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Iteration Scheme for Random Common Fixed Points of Two Random Asymptotically Nonexpansive Random Operators

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***Abstract* — In this chapter, We prove the existence of a common random fixed point of two asymptotically nonexpansive random operators through strong and weak convergences of an iterative process. The necessary and sufficient condition for the convergence of sequence of measurable functions to a random fixed point of asymptotically quasi-nonexpansive random operators in uniformly convex Banach spaces is also established. Our random iteration scheme includes Ishikawa type and Mann type random iterations as special cases. The results obtained in this chapter represent an extension as well as refinement of previous known results. A new random iterative scheme for approximating random common fixed points of two random asymptotically nonexpansive random operators are defined and we have proved weak and strong convergence theorems in a uniformly convex Banach space.**

***Keyword* — Two-step random iteration, Random asymptotically random operator, Banach space, Strong convergence, Common fixed point.**

1. INTRODUCTION

The study of random fixed points has been an active area of contemporary research in mathematics. Some of the works in this field are noted in [2, 19, 4, 5, 44]. In particular, random iteration schemes leading to random fixed points were introduced in [19]. After that, random iterations for getting solutions of random operator equations and fixed points of random operators have been discussed, as, for example, in [4, 5]. The aim of this paper is to define an iteration scheme for two random operators on a nonempty closed convex subset of a separable Hilbert space and consider its convergence to a common random fixed point of two random operators. The two random operators satisfy some contractive inequality. Contractive mappings have often been subjects of fixed point studies. For a review of the subject matter, we refer to [43].

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is needed for the study of various classes of random equations. The study of random fixed point theory was initiated by the Prague school of probabilists in the 1950s [45, 46, 8]. Common random fixed point theorems are stochastic generalization of classical common fixed point theorems. The machinery of random fixed point theory provides a convenient way of modeling many problems arising from economic theory, see for example [9] and references mentioned therein. Random methods have revolutionized the financial markets. The survey article by Bharucha-Reid [22, 11] attracted the attention of several mathematicians and gave wings to this theory. Itoh [12] extended Spaceks and Hanss theorem to multi-valued contraction mappings. Now this theory has become the full edged research area and various ideas associated with random fixed point theory are used to obtain the solution of nonlinear random system (see [20, 21, 22]). Papageorgiou [13, 14], Beg [15, 16] studied common random fixed points and random coincidence points of a pair of compatible random operators and proved fixed point theorems for contractive random operators in Polish spaces. Recently, Beg and Shahzad [18], Choudhury [19], and Badshah and Sayyed [17] used different iteration processes to obtain common random fixed points. The aim of this paper is to find common random fixed points of two asymptotically nonexpansive random operators through strong as well as weak convergences of sequence of measurable functions in the setup of uniformly convex Banach spaces. We construct different random iterative algorithms for asymptotically quasi-nonexpansive random operators on an arbitrary Banach space and establish their convergence to random fixed point of the operators mentioned afore. This class of asymptotically nonexpansive mappings was to introduced by Goebel and Kirk [31] in 1972. They proved that, if K is a nonempty bounded closed convex subset of a uniformly convex Banach space E, then every asymptotically nonexpansive self-mapping T of K has a fixed point. The fixed point iteration process for asymptotically nonexpansive mapping in Banach spaces including Mann and Ishikawa iterations processes have been studied extensively by many authors; see ([25]-[42]).

Current Trends in Technology and Science
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2. PRELIMINARIES

Following are the definitions and lemma used to prove the results in the next section.

Definition 1.1. Let K be a nonempty closed convex subset of a uniformly convex separable Banach space E. A mapping $T:K\to K$ is said to be random asymptotically nonexpansive on K if there exists a sequence k_n, $k_n \geq 1$ with $\lim_{b\to\infty} k_n = 1$, such that

$$\|T^n(\omega,x) - T^n(\omega,y)\| \leq k_n \|x-y\|$$

for each $\omega \in \Omega$ and each $n \geq 1$. If $k_n \geq 1$, then T is known as a random nonexpansive random operator.

Definition 1.1. A mapping $T:K\to K$ is uniformly k-Lipschitzian if for some K > 1 is said to be random asymptotically nonexpansive on K if there exists a sequence k_n, $k_n \geq 1$ with $\lim_{b\to\infty} k_n = 1$, such that

$$\|T^n(\omega,x) - T^n(\omega,y)\| \leq k\|x-y\|$$

and for each $\omega \in \Omega$.

Definition 1.3 . Let E be a uniformly convex separable Banach space, K be a nonempty closed convex subset of E, and $T:K\to K$ be a random asymptotically nonexpansive random operator. Then I -T is said to be demi-closed at 0, if $x_n(\omega) \to x$ converges weakly and $x_n(\omega) - T(\omega, x_n(\omega)) \to 0$ converges strongly, then it is implies that $x \in K$ and $T(\omega; x) = x$.

Definition 1.4 [30] . Suppose two mappings $S, T:K\to K$, where K is a subset of a normed space E, said to be satisfy condition (A') if there exists a nondecreasing function $F: [0, \infty) \to [0, \infty)$ with $F(0) = 0$, $f(r) > 0$ for all $r \in (0,\infty)$ such that either

$$\|x - Sx\| \geq f(d(x,F)) \text{ or } \|x - Tx\| \geq \_f(d(x,F))$$

for all $x \in K$ where

$$d(x, F) = \inf \{\|x - p\| : p \in F = F(S) \cap F(T)\}.$$

Several authors have been studied weak and strong convergence problems of iterative sequence (with errors) for asymptotically nonexpansive type mappings in a Hilbert space or a Banach space (see [25],[34], [35],[37]).

In 2007, Agrawal et al. [1] introduced the following iteration process:

$$\begin{cases} x_1 = x \in K \\ x_{n+1} = (1-\alpha_n)T^n x_n + \alpha_n T^n y_n \\ y_n = (1-\beta_n)x_n + \beta_n T^n x_n,\ n \in N, \end{cases} \tag{1.1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in (0,1). They showed that this process converges at a rate same as that Picard iteration and faster than Mann for contractions.

The above process deals with one mapping only. The case of two mappings in iterative processes has also remained under study since Das and Debtat [26] gave and studied a two mappings process. Also see, for example [33] and [39]. The problem of approximating common fixed points of finitely many mapping plays an important role in applied mathematics especially in the theory of evaluation equation and the minimization problems. See ([27], [28],[29], [40]) for example.

In 2001, Khan and Takahashi [33] approximated the fixed points of two asymptotically nonexpansive mappings $S, T:K\to K$ through the sequence $\{x_n\}$ given by

$$\begin{cases} x_1 = x \in K \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n S^n y_n \\ y_n = (1-\beta_n)x_n + \beta_n T^n x_n,\ n \in N, \end{cases} \tag{1.2}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in (0,1).

Recently, Khan et al. [32] modi_ed the iteration process (1.2) to the case of two mappings as follows:

$$\begin{cases} x_1 = x \in K \\ x_{n+1} = (1-\alpha_n)T^n x_n + \alpha_n S^n y_n \\ y_n = (1-\beta_n)x_n + \beta_n T^n x_n,\ n \in N, \end{cases} \tag{1.3}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in (0,1).

In this paper, we introduced a new implicit random iteration scheme as below:

Definition 1.5 Let $S, T:\Omega \times K \to X$ be two random operators, where K is a nonempty convex subset of a real uniformly convex separable Banach space X. The random iteration scheme is defined as follows:

$$\begin{cases} \xi_{n+1}(\omega) = \alpha_n \xi_n(\omega) + \beta_n T^n(\omega, \xi_n(\omega)) + \gamma_n S^n(\omega, \eta_n(\omega)) \\ \eta_n(\omega) = \alpha'_n \xi_n(\omega) + \beta'_n S^n(\omega, \xi_n(\omega)) + \gamma'_n T^n(\omega, \xi_n(\omega)),\ n\ N, \end{cases} \tag{1.4}$$

for each $\omega \in \Omega$, where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\alpha'_n\}$, $\{\beta'_n\}$ and $\{\gamma'_n\}$ are sequence in [0, 1] and $\alpha_n + \beta_n + \gamma_n = \alpha'_n + \beta'_n + \gamma'_n = 1$, for fixed points of random asymptotically nonexpansive random operator T in uniformly convex separable Banach space.

Observe that in (1.4) if we set T = I, then the scheme will reduce to:

Definition 1.6 Let $S, T:\Omega \times K \to X$ be two random operators, where K is a nonempty convex subset of a real uniformly convex separable Banach space X. Then

Current Trends in Technology and Science
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$$\begin{cases}\xi_{n+1}(\omega)=(1-\gamma_n)\xi_n(\omega)+\gamma_n)S^n(\omega,\eta_n(\omega)) \\ \eta_n(\omega)=(1-\beta_n^{'})\xi_n(\omega)+\beta_n^{'}S^n(\omega,\xi_n(\omega)),\ n\ N,\end{cases} \quad (1.5)$$

for each $\omega \in \Omega$, where $\{\gamma_n\}$, $\{\beta_n^{'}\}$ are sequence in [0, 1].

Similarly, in (1.5) if we set S = T, $\gamma_n = \alpha_n$ and $\beta_n^{'} = 0$, then the scheme will reduce to the Mann type random iteration scheme given by:

Definition 1.7 Let $S, T: \Omega \times K \to X$` be two random operators, where K is a nonempty convex subset of a real uniformly convex separable Banach space X. Then Mann iteration process is defined by

$$\xi_{n+1}(\omega)=(1-\alpha_n)\xi_n(\omega)+\alpha_n T^n(\omega,\xi_n(\omega)),\ n\ N, \quad (1.6)$$

for each $\omega \in \Omega$, where $\{\gamma_n\}$, $\{\beta_n^{'}\}$ are sequence in [0, 1].

Remark 1.1 Let F be a closed and convex subset of a separable Banach space X and the sequence of functions $\{\xi_n\}$ defined as in Definition 1.5 is point-wise convergent, that is, $\xi_n(\omega) \to q := \xi(\omega)$, for each $\omega \in \Omega$. Then closedness of F implies that is a mapping from to F. Since F is a subset of a separable Banach space X, so if T is a continuous random operator, then by ([24], Lemma 8.2.3), the map $\omega \to T^n(\omega,\xi(\omega))$ is a measurable function for any measurable function f from to F. Hence f_g is a sequence of measurable functions and $\omega: \Omega \to F$ being a limit of a sequence of measurable functions is also measurable.

Lemma 1.1 ([41], Lemma 1) : Let $\{a_n\},\{b_n\}$ and $\{\delta_n\}$ be sequence of nonnegative real numbers satisfying the inequality

$$a_{n+1} \le (1+\delta_n)a_n + b_n.$$

If $\sum_{n=1}^{\infty}\delta_n < \infty$ and $\sum_{n=1}^{\infty}\delta_n < \infty$, then $\lim_{n\to\infty} a_n$ exists. In particular, if $\{a_n\}$ has a subsequence converging to 0, then $\lim_{n\to\infty} a_n = 0$.

Lemma 1.2 [37]] Suppose that E be a uniformly convex Banach space and $0 < p \le t_n \le q < 1$ for all $n \in N$. Let $\{x_n\}$ and $\{y_n\}$ be two sequences of E such that

$$\limsup_{n\to\infty} \| x_n \| \le r, \ \limsup_{n\to\infty} \| y_n \| \le r$$

and $\lim_{n\to\infty} \| t_n x_n + (1-t_n)y_n = r$ hold for some $r \ge 0$. Then

$$\lim_{n\to\infty} \| x_n - y_n \| = 0.$$

Lemma 1.3 [23] Let F be a nonempty, closed, bounded, and convex subset of a uniformly convex separable Banach space X satisfying the Opial condition. Let T be an asymptotically nonexpansive mapping of F into itself. Then I - T is demiclosed with respect to 0.

Let (Ω,Σ) be a measurable space $(\Sigma$ - sigmaalgebra$)$ and let F be a nonempty subset of a Banach space X. We will denote by K(X) the family of all compact subsets of X with Hausdorff metric H induced by the metric of X. A mapping $\xi: \Omega \to X$ is measurable if $\xi \in (U)\Sigma$, for each open subset U of X. The mapping $T: \Omega \times X \to X$ is a random map if and only if for each fixed $x \in X$, the mapping $T(., x): \Omega \to X$ is measurable and it is continuous if for each $\omega \in \Omega$, the mapping $T(\omega,.): X \to X$ is continuous. A measurable mapping $\xi: \Omega \to X$ is a random fixed point of a random map $T: \Omega \times X \to X$ if and only if $T(\omega,\xi(\omega)) = \xi(\omega)$, for each $\omega \in \Omega$. We denote the set of random fixed points of a random map T by F(T). Let $B(x_0, r)$ denote the spherical ball centered at x0 with radius r, defined as the set $\{x \in X: \| x_n - x_0 \| \le r\}$.

We denote the n^{th} iterate $T(\omega, T(\omega, T(\omega, ..., T(\omega, x)...)))$ of T by $T_n(\omega, x)$. The letter I denotes the random mapping $I: \Omega! \times X \to X$ defined by $I(\omega, x) = x$ and $T^0 = I$.

Let us now gather some pre-requisites. Let $X = \{x \in E: \| x \| = 1\}$ and E^* be the dual of E. The space E has :

(i) Gateaux differentiable norm if $\lim_{n\to\infty} | \frac{\| x + ty \| - \| x \|}{t}$, exists for each $x, y \in S$.

(ii) Frechet differentiable norm (see e.g. [38]) for each x in S, the above limit exists and is attained uniformly for y in S and in this case, it is also well-known that's

$$\langle h, J(x)\rangle + \frac{1}{2}\| x \|^2 \le \frac{1}{2}\| x + h \|^2 \le \langle h, J(x)\rangle + \frac{1}{2}\| x \|^2 + b(\| h \|)$$

for all $x, h \in E$, where J is the Frechet derivative of the function $\frac{1}{2}\| . \|^2$, $x \in E, \langle .,. \rangle$ is the dual pairing between E and E*, and b is an increasing function defined on $[0, \infty)$ such that $\lim_{t\to 0}\frac{b(t)}{t} = 0$

Current Trends in Technology and Science
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(iii) Opial's condition [36] if for any sequence $\{x_n\}$ in E, $x_n \xrightarrow{weak} x$ implies that $\limsup_{n\to\infty} \| x_n - x \| < \limsup_{n\to\infty} \| x_n - y \|$ for all $y \in E$ with $y \neq x$.

3. MAIN RESULTS

In this section, we have proved common random fixed point of two asymptotically nonexpansive random operators. This purpose, we construct a sequence of measurable functions and consider its strong as well as weak convergence to a common random fixed point of random operators mentioned afore in the framework of uniformly convex Banach spaces.

Theorem 3.1 : Let X be a real uniformly convex separable Banach space, K a nonempty closed convex subset of X. Suppose that $S, T : \Omega \times K \to K$ is a non-self asymptotically nonexpansive random mapping with a measurable mapping sequence $k_n : \Omega \to [1.+\infty)$, $\sum_{n=1}^{\infty} (k_n(\omega) - 1) < \infty$, $k_n(\omega) \to 1$ as $n \to \infty$ for each $\omega \in \Omega$. Assume that $F(T) \neq \phi$ and the sequence $\{\xi_n\}$ defined by (1.4), then $\lim_{n\to\infty} \| \xi_n(\omega) - \xi^*(\omega) \|$ exists for each $\omega \in \Omega$ and any $\xi^* \in F(T)$.

Proof :Let $\xi^* \in F(T)$ for each $\omega \in \Omega$ and From (1.4), we conclude that

$$\begin{aligned}
&\| \xi_{n+1}(\omega) - \xi^*(\omega) \| \\
&= \| \alpha_n \xi_n(\omega) + \beta_n T^n(\omega, \xi_n(\omega)) \\
&\quad + \gamma_n S^n(\omega, \eta_n(\omega)) - \xi^*(\omega) \| \\
&= \| (\alpha_n \xi_n(\omega) - \xi^*(\omega)) + (\beta_n T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)) \\
&\quad + (\gamma_n S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)) \| \\
&\leq \alpha_n \| \xi_n(\omega) - \xi^*(\omega) \| + \beta_n k_n \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\quad + \gamma_n k_n \| \eta_n(\omega) - \xi^*(\omega) \| \\
&\leq \alpha_n \| \xi_n(\omega) - \xi^*(\omega) \| + \beta_n k_n \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\quad + \gamma_n k_n [\alpha'_n \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\quad + \beta'_n \| S^n(\omega, \xi_n(\omega)) - \xi^*(\omega) \| \\
&\quad + \gamma'_n \| T^n(\omega, \xi_n(\omega)) - \xi^*(\omega) \|] \\
&\leq \alpha_n \| \xi_n(\omega) - \xi^*(\omega) \| + \beta_n k_n \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\quad + \gamma_n k_n \alpha'_n \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\quad + \gamma_n k_n^2 \beta'_n \| \xi_n(\omega) - \xi^*(\omega) \| + \gamma_n k_n^2 \gamma'_n \| \xi_n(\omega)) - \xi^*(\omega) \| \\
&\leq (\alpha_n + \beta_n k_n + \gamma_n k_n \alpha'_n + \gamma_n k_n^2 \beta'_n \\
&\quad + \gamma_n k_n^2 \gamma'_n) \| \xi_n(\omega) - \xi^*(\omega) \| \\
&\leq (1 + \{1 - \alpha_n + \gamma_n k_n (1 - \alpha'_n)\}(k_n - 1) \| \xi_n(\omega) - \xi^*(\omega) \|]
\end{aligned}$$

Since and $\sum_{n=1}^{\infty} (k_n(\omega) - 1) < \infty$ as $n \to \infty$, consequently, the condition of Theorem 3.1 follows from Lemma 1.1. This completes the proof.

Theorem 3.2 Let E be a uniformly convex separable Banach space and K be a nonemptyclosed convex subset of E. Let $S, T : \Omega \times K \to K$ be two random asymptotically non-expansive random operators of K with $\sum_{n=1}^{\infty} (k_n(\omega) - 1) < \infty$ satisfying the iteration process (1.4). Le $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$, $\{\alpha'_n\}, \{\beta'_n\}$ and $\{\gamma'_n\}$ are sequence in [0, 1] such that $\alpha_n + \beta_n + \gamma_n = \alpha'_n + \beta'_n + \gamma'_n = 1$.
If $F(T) \neq \phi$, $k_n(\omega) \to 1$ as $n \to \infty$, then

$$\lim_{n\to\infty} \| \xi_n(\omega) - T(\omega, \xi_n(\omega)) \| = \lim_{n\to\infty} \| \xi_n(\omega) - S(\omega, \xi_n(\omega)) \| = 0 .$$

Proof : Let $\xi^* \in F(T)$.

Current Trends in Technology and Science
ISSN : 2279-0535. Volume : 04, Issue : 06 (Oct- Nov 2015)

Since $\lim_{n\to\infty} \|\xi_n(\omega) - \xi^*(\omega)\|$ exists as proved in Theorem3.1. Suppose $\lim_{n\to\infty} \|\xi_n(\omega) - \xi^*(\omega)\| = c$; where $c \geq 0$ is a real number. Assume that c > 0. Now,

$$\|\eta_n(\omega) - \xi^*(\omega)\|$$
$$= \|\alpha'_n \xi_n(\omega) + \beta'_n S^n(\omega, \xi_n(\omega)) + \gamma'_n T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|$$
$$= \alpha'_n \|\xi_n(\omega) - \xi^*(\omega)\| + \beta'_n \|S^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| + \gamma'_n \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|$$
$$\leq \alpha'_n \|\xi_n(\omega) - \xi^*(\omega)\| + \beta'_n k_n \|\xi_n(\omega) - \xi^*(\omega)\| + \gamma'_n k_n \|\xi_n(\omega) - \xi^*(\omega)\|$$
$$\leq (\alpha'_n + \beta'_n k_n + \gamma'_n k_n) \|\xi_n(\omega) - \xi^*(\omega)\|$$
$$\leq [1 + \alpha'_n (k_n - 1)] \|\xi_n(\omega) - \xi^*(\omega)\|$$

Taking limsup on both sides of the above inequality, we get

$$\limsup_{n\to\infty} \|\eta_n(\omega) - \xi^*(\omega)\| \leq c. \qquad (3.1)$$

Also

$$\|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| \leq k_n \|\xi_n(\omega) - \xi^*(\omega)\|$$

for all n = 1, 2, …, so

$$\|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| \leq c \qquad (3.2)$$

Next, we consider

$$\|S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\| \leq k_n \|\eta_n(\omega) - \xi^*(\omega)\|$$

Taking limsup on both sides of the above inequality and using (3.1), we have

$$\|S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\| \leq c$$

Furthermore,

$$c = \lim_{n\to\infty} \|\xi_{n+1}(\omega) - \xi^*(\omega)\|$$
$$= \lim_{n\to\infty} \alpha_n \|\xi_n(\omega) - \xi^*(\omega)\| + \beta_n \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| + \gamma_n \|S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\|$$
$$= (1 - \gamma_n) |T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| + \gamma_n \|S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\|$$

by Lemma 1.2, we get

$$\lim_{n\to\infty} \|T^n(\omega, \xi_n(\omega)) - S^n(\omega, \eta_n(\omega))\| = 0 \quad (3.3)$$

Now,

$$\|\xi_{n+1}(\omega) - \xi^*(\omega)\|$$
$$= \|\alpha_n \xi_n(\omega) + \beta_n T^n(\omega, \xi_n(\omega)) + \gamma_n S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\|$$
$$\leq \|(1 - \gamma_n) T^n(\omega, \xi_n(\omega)) + \gamma_n S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)|$$
$$\leq \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|\| + \gamma_n \|T^n(\omega, \xi_n(\omega)) - S^n(\omega, \eta_n(\omega))\|$$

which yields that

$$c \leq \liminf_{n\to\infty} \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|.$$

So that from (3.2) gives

$$\lim_{n\to\infty} \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\| = c.$$

In turn,

$$\|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|$$
$$\leq \|T^n(\omega, \xi_n(\omega)) - S^n(\omega, \eta_n(\omega))\| + \|S^n(\omega, \eta_n(\omega)) - \xi^*(\omega)\|$$
$$\leq \|T^n(\omega, \xi_n(\omega)) - S^n(\omega, \eta_n(\omega))\| + k_n \|\eta_n(\omega) - \xi^*(\omega)\|$$

which implies that

$$c \leq \liminf_{n\to\infty} \|\eta_n(\omega) - \xi^*(\omega)\|. \qquad (3.4)$$

By (3.1) and (3.4), we have

$$\lim_{n\to\infty} \|\eta_n(\omega) - \xi^*(\omega)\| = c. \qquad (3.5)$$

Again, we get

$$c = \lim_{n\to\infty} \|\eta_n(\omega) - \xi^*(\omega)\|$$
$$= \lim_{n\to\infty} (1 - \gamma'_n) \|\xi_n(\omega) - \xi^*(\omega)\| + \gamma'_n \|T^n(\omega, \xi_n(\omega)) - \xi^*(\omega)\|$$

gives by Lemma 1.2 that

$$\lim_{n\to\infty} \|T^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\| = 0. \qquad (3.6)$$

Notice that

$$\|\eta_n(\omega) - \xi_n(\omega)\| = \gamma'_n \|T^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\|.$$

Hence by (3.6)

$$\lim_{n\to\infty} \|\eta_n(\omega) - \xi_n(\omega)\| = 0 \qquad (3.7)$$

Now

Current Trends in Technology and Science
ISSN : 2279-0535. Volume : 04, Issue : 06 (Oct- Nov 2015)

$$\|\xi_{n+1}(\omega)-\xi_n(\omega)\|$$
$$=\|\alpha_n\xi_n(\omega)+\beta_n T^n(\omega,\xi_n(\omega))$$
$$+\gamma_n S^n(\omega,\eta_n(\omega))-\xi_n(\omega)\|$$
$$\le(1-\alpha_n)\|T^n(\omega,\xi_n(\omega))-\xi_n(\omega)\|$$
$$+\gamma_n\|T^n(\omega,\xi_n(\omega))-S^n(\omega,\eta_n(\omega))\|$$

This gives

$$\lim_{n\to\infty}\|\xi_{n+1}(\omega)-\xi_n(\omega)\|=0. \qquad (3.8)$$

so that

$$\|\xi_{n+1}(\omega)-\eta_n(\omega)\|\le|\xi_{n+1}(\omega)-\xi_n(\omega)\|$$
$$+\|\xi_n(\omega)-\eta_n(\omega)\|\to 0 \text{ as } n\to 0$$
.

This gives

$$\lim_{n\to 0}\|\xi_{n+1}(\omega)-\eta_n(\omega)\|=0. \qquad (3.9)$$

Moreover, from

$$\|\xi_{n+1}(\omega)-S^n(\omega,\eta_n(\omega))\le\|\xi_{n+1}(\omega)-\xi_n(\omega)\|$$
$$+\|\xi_n(\omega)-T^n(\omega,\xi_n(\omega))\|$$
$$+\|T^n(\omega,\xi_n(\omega))-S^n(\omega,\eta_n(\omega)\|$$

which is gives that

$$\lim_{n\to\infty}\|\xi_{n+1}(\omega)-S^n(\omega,\eta_n(\omega))=0.$$

Using (3.3), (3.6) and (3.7) we obtained

$$\|\xi_n(\omega)-S^n(\omega,\xi_n(\omega))\|$$
$$\le\|\xi_n(\omega)-T^n\xi_n(\omega))\|$$
$$+\|T^n(\omega,\xi_n(\omega))-S^n(\omega,\eta_n(\omega))\|$$
$$+\|S^n(\omega,\eta_n(\omega)-S^n(\omega,\xi_n(\omega))\|$$
$$\le\|\xi_n(\omega)-T^n\xi_n(\omega))\|$$
$$+\|T^n(\omega,\xi_n(\omega))-S^n(\omega,\eta_n(\omega))\|$$
$$+k_n\|\eta_n(\omega)-\xi_n(\omega)\|$$

gives that

$$\lim_{n\to\infty}\|\xi_n(\omega)-S^n(\omega,\xi_n(\omega))\|=0,$$

and

$$\|\xi_{n+1}(\omega)-S(\omega,\xi_{n+1}(\omega))\|$$
$$\le\|\xi_{n+1}(\omega)-S^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+\|S^{n+1}(\omega,\xi_{n+1}(\omega))|S(\omega,\xi_{n+1}(\omega))\|$$
$$\le\|\xi_{n+1}(\omega)-S^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+k^n\|S^n(\omega,\xi_{n+1}(\omega))-\xi_{n+1}(\omega)\|$$
$$\le\|\xi_{n+1}(\omega)-S^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+k^n[\|S^n(\omega,\xi_{n+1}(\omega))-S^n(\omega,\eta_n(\omega))\|$$
$$+\|S^n(\omega,\eta_n(\omega))-\xi_{n+1}(\omega)\|]$$
$$\le\|\xi_{n+1}(\omega)-S^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+k_n^2\|\xi_{n+1}(\omega)-\eta_n(\omega)\|$$
$$+k^n\|S^n(\omega,\eta_n(\omega))-\xi_{n+1}(\omega)\|$$

this implies that

$$\lim_{n\to\infty}\|\xi_{n+1}(\omega)-S(\omega,\xi_{n+1}(\omega))\|=0.$$

Now

$$\|\xi_{n+1}(\omega)-T(\omega,\xi_{n+1}(\omega))\|$$
$$\le\|\xi_{n+1}(\omega)-T^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+\|T^{n+1}(\omega,\xi_{n+1}(\omega))-T^{n+1}(\omega,\xi_n(\omega))\|$$
$$+\|T^{n+1}(\omega,\xi_n(\omega))|-T(\omega,\xi_{n+1}(\omega))\|$$
$$\le\|\xi_{n+1}(\omega)-T^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+k_n\|\xi_{n+1}(\omega)-\xi_n(\omega)\|$$
$$+k_n\|T^n(\omega,\xi_n(\omega))-\xi_{n+1}(\omega)\|$$
$$=\|\xi_{n+1}(\omega)-T^{n+1}(\omega,\xi_{n+1}(\omega))\|$$
$$+k_n\|\xi_{n+1}(\omega)-\xi_n(\omega)\|$$
$$+k_n\alpha_n\|T^n(\omega,\xi_n(\omega))-S^n(\omega,\eta_n(\omega))\|$$

which is yields

$$\lim_{n\to\infty}\|\xi_{n+1}(\omega)-T(\omega,\xi_{n+1}(\omega))\|=0.$$

This completes the proof.

Theorem 3.3 Let E be a uniformly convex Banach space satisfying Opial condition and K; T; S and $\{\xi_n(\omega)\}$ be taken as Theorem 3.2. If $F(S)\cap F(T)\ne\phi$; I-T and I-S are demiclosded at zero, then $\{\xi_n(\omega)\}$ converges weakly to a common fixed point of S and T.

Proof :Let $F(S)\cap F(T)\ne\phi$. Then as proved in Theorem 3.1. Let $\xi:\Omega\to F$ be the common random fixed point of S and T. Let ϕ and φ be two functions from Ω to F and let $\{\xi_i(\omega)\}$ and $\{\varphi_j(\omega)\}$ be two subsequences of $\{\xi_n(\omega)\}$ such that $\{\xi_i(\omega)\}$ and

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$\{\xi_j(\omega)\}$ converge weakly to $\phi(\omega)$ and $\varphi(\omega)$, for each $\omega \in \Omega$ as $m, k \to \infty$. As proved in Theorem 3.1, $\lim_{n\to\infty} \|\xi_{n+1}(\omega) - \xi(\omega)\|$ exists, for each $\omega \in \Omega$ and S and T are asymptotically nonexpansive, $I - S(\omega)$ and $I - T(\omega)$ are demiclosed with respect to 0, by Lemma 1.3, it follows that $S(\omega, \phi(\omega)) = \phi(\omega)$ and $S(\omega, \phi(\omega)) = \phi(\omega)$. Now measurability of $\phi(\omega)$ follows from the fact that S and T are completely continuous random operators and from Remark 1.1. Thus $\phi(\omega)$ is a common random fixed point of S and T. Similarly, it can be shown that . is also a common random fixed point of random operators T and S. Now we prove that $\phi(\omega) = \varphi(\omega)$, for every $\omega \in \Omega$. If not so, then for some $\omega \in \Omega$, $\phi(\omega) \neq \varphi(\omega)$. Now by Opialfs condition

$$\begin{aligned}\lim_{n\to\infty} \|\xi_n(\omega) - \phi(\omega)\| &\le \lim_{n_i\to\infty} \|\xi_{n_i}(\omega) - \phi(\omega)\| \\ &< \lim_{n_i\to\infty} \|\xi_{n_i}(\omega) - \varphi(\omega)\| \\ &= \lim_{n\to\infty} \|\xi_n(\omega) - \varphi(\omega)| \\ &= \lim_{n_j\to\infty} \|\xi_{n_j}(\omega) - \varphi(\omega)\| \\ &< \lim_{n_j\to\infty} \|\xi_{n_j}(\omega) - \phi(\omega)\| \\ &= \lim_{n\to\infty} \|\xi_n(\omega) - \phi(\omega)\|.\end{aligned}$$

This is a contradiction so $\phi(\omega) = \varphi(\omega)$. Hence $\{\xi_n(\omega)\}$ converges weakly to a common fixed point of S and T.

Theorem 3.4 Suppose X be a uniformly convex separable Banach space and K, S, T, $\{\xi_n(\omega)\}$ be same as in Theorem 3.1. If $F_1 \neq \phi$ and S, T satisfy the Condition (A'), then

1. F_1 is a closed set.
2. $\{\xi_n(\omega)\}$ g converges to a common random fixed point of S and T.

Proof. Step : 1. Let $\{\xi_n(\omega)\} \subset F_1$ be such that $\{\xi_n(\omega)\} \to p(\omega)$ as $n \to \infty$, for each $\omega \in \Omega$. Since $\{\xi_n(\omega)\}$ is a measurable sequence, the limit x of $\{\xi_n(\omega)\}$ is measurable, too. In addition

$$\begin{aligned}\|T(\omega, p(\omega)) - p(\omega)\| &\le \|T(\omega, p(\omega)) - T(\omega, \xi_n(gw)) \\ &\quad + \xi_n(\omega) - p(\omega)\| \\ &\le 2\|\xi_n(\omega) - p(\omega)\|.\end{aligned}$$

This implies that $T(\omega, p(\omega)) = p(\omega)$, for each $\omega \in \Omega$, therefore, $p(\omega) \in F(T)$. Similarly, we have $p(\omega) \in F(S)$. Thus, $p(\omega)$ is a common random fixed point of S and T. Hence F1 is a closed set.

Step : 2. For any $q \in F_1$, by Theorem 3.1, for each $\omega \in \Omega$ $\lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\|$ exists. Let it be c for some $c \ge 0$, by Theorem 3.2,

$$\lim_{n\to\infty} \|\xi_n(\omega) - T(\omega, \xi_n(\omega)) = \lim_{n\to\infty} \|\xi_n(\omega) - S(\omega, \xi_n(\omega)) = 0,$$

for each $\omega \in \Omega$. Moreover, $\|\xi_{n+1}(\omega) - q(\omega)\| \le \|\xi_n(\omega) - q(\omega)\|$ that

$$\inf_{q\in F_1} \|\xi_{n+1}(\omega) - q(\omega)\| \le \inf_{q\in F_1} \|\xi_n(\omega) - q(\omega)\|.$$

That implies that

$0 \le d(\xi_{n+1}(\omega), F_1) \le d(\xi_n(\omega), F_1)$.

Thus $\lim_{n\to\infty} d(\xi(\omega), F_1)$ exists, for each $\omega \in \Omega$. Since S, T satisfy Condition (A'), we have

$$\frac{1}{2}(\|\xi_n(\omega), T(\omega, \xi(\omega))\| + |\xi_n(\omega), S(\omega, \xi(\omega))\| \ge d(\xi(\omega), F_1),$$

we have $\lim_{n\to\infty} f(d(\xi(\omega), F_1)) \le 0$. Because f is a increasing function with f(0) = 0, so that $\lim_{n\to\infty} f(d(\xi(\omega), F_1)) = 0$, for each $\omega \in \Omega$. For any $\varepsilon > 0$ and any given $\omega \in \Omega$, since $\lim_{n\to\infty} f(d(\xi(\omega), F_1)) = 0$, there exists natural number n_1 such that when $n \ge n_1$, $d(\xi(\omega), F_1) < \frac{\varepsilon}{3}$.Thus, there exists $p(\omega) \in F_1$ such that for above ε and ω there exists positive integer $N_1 \ge n_1$, $\|\xi_n(\omega) - p(\omega)\| < \frac{\varepsilon}{2}$.

Now for arbitrary $n, m \in N_1$, consider

$$\|\xi_n(\omega) - \xi_m(\omega)\| \le \|\xi_n(\omega) - p(\omega)\| + \|\xi_m(\omega) - p(\omega)\| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

It implies that $\{\xi_n(\omega)\}$ is a Cauchy sequence in K. Let $\xi_n(\omega) = p(\omega)$. Since $\{\xi_n(\omega)\}$ is a sequence of measurable mappings, so p, the limit of the sequence of measurable mapping $\{\xi_n(\omega)\}$, is measurable, too. In addition, it follows from Lemma 1.3 and Theorem 3.2 that $T(\omega, p(\omega)) = p(\omega) = S(\omega, p(\omega))$. Therefore, p is a common random fixed point of S and T. Thus, by Theorem 3.1, $\lim_{n\to\infty} \|\xi_n(\omega) - p(\omega)\| = 0$, for each $\omega \in \Omega$. This implies that $\{\xi_n(\omega)\}$ converges to the common random fixed point p of S and T. The proof is completed.

Theorem 3.5 Let X be a uniformly convex separable Banach space and K, S, T, $\{\xi_n(\omega)\}$ be same as in Theorem 3.1. If $F_1 \neq \phi$, one of S and T is completely continuous random operator, then $\{\xi_n(\omega)\}$ converges to a common random fixed point of S and T.

Proof. For any $p(\omega) \in F_1$, by Theorem 3.1, for each $\omega \in \Omega$ $\lim_{n\to\infty} \| \xi_n(\omega) - \xi(\omega) \|$ exists. By the condition of Theorem 3.5, we may assume that T is completely continuous random operator. Since, $\{\xi_n(\omega)\}$ is a bounded sequence and X is uniformly convex separable Banach space, there exists convergent subsequence $\{\xi_{n_k}(\omega)\}$ of $\{\xi_n(\omega)\}$ such that $\{T(\omega, \xi_{n_k}(\omega))\}$ is convergent, for each $\omega \in \Omega$. Suppose $\{\xi_{n_k}(\omega)\} \to \xi(\omega)$ for each $\omega \in \Omega$, then $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. The mapping $\xi : \Omega \to K$ being a point-wise limit of the measurable mappings $\{\xi_n(\omega)\}$ is measurable, by the continuity of T. That is $\xi(\omega) \in F(T)$. Since $\| (\xi_n(\omega), S(\omega, \xi_n(\omega)) \| = 0$ and $\xi_{n_k}(\omega) \to \xi(\omega)$ as $k \to \infty$, we obtain $S(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. So $\xi(\omega) \in F(S)$. Hence $\xi(\omega) \in F_1$. Since $\lim_{n\to\infty} \| \xi_n(\omega) - \xi(\omega) \|$ exists and $\lim_{n\to\infty} \| \xi_{n_k}(\omega) - \xi(\omega) \| = 0$, for each $\omega \in \Omega$. Therefore,

$$\lim_{n\to\infty} \| \xi_n(\omega) - \xi(\omega) \| = 0,$$

for each $\omega \in \Omega$. The proof is completed.

Corollary 3.1 . Let K be a nonempty closed convex subset of a uniformly convex separable Banach space E. Suppose T be a random asymptotically nonexpansive random operator of K. Let $\{\xi_n(\omega)\}$ be defined by the iteration (1.5), where $\{\gamma_n\}$ and $\{\beta'_n\}$ in [0, 1] for all $n \in N$, then $\{\xi_n(\omega)\}$ converges strongly to a fixed point of T.

Proof. Suppose S = T in the above theorem.

Corollary 3.2 . Let K be a nonempty closed convex subset of a uniformly convex separable Banach space E. Suppose T be a random asymptotically nonexpansive random operator of K. Let $\{\xi_n(\omega)\}$ be defined by the iteration (1.6), where $\{\gamma_n\}$ and $\{\beta'_n\}$ in [0, 1] for all $n \in N$, then $\{\xi_n(\omega)\}$ converges strongly to a fixed point of T.

Proof. Suppose T=I in the above theorem.

Corollary 3.3. Suppose E be a separable Banach space satisfying Opial condition and let K and T be taken as in (3.2). Let $F(T) \neq \phi$. Now if the mapping I -T is demiclosed at zero, then $\{\xi_n(\omega)\}$ defined by (1.1) converges weakly to a fixed point of T.

Corollary 3.4 . Let E be a uniformly convex separable Banach space which has a Frechet di_erentiable norm and let K and T be taken as theorem 3.2. Let $F(T) \neq \phi$ taken $\{\xi_n(\omega)\}$ defined by (1.6) converges weakly to a fixed point of T.

Corollary 3.5 . Let E be a uniformly convex separable Banach space satisfying Opial condition and let K and T be taken as in Theorem 3.2. Let $F(T) \neq \phi$. If the mapping I-T is demiclosed at zero, then $\{\xi_n(\omega)\}$ defined by (1.6) converges weakly to a fixed point of T.

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