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Estimating Areal Rainfall by Use of Galerkin's Method (Case study; Mashhad Plain Basin)

M. Irani
Department of Civil Engineering, Islamic Azad University of Mashhad, Iran,
Email ID: mojtaba_ir64@yahoo.com

F. Khamchinmoghadam
Department of Civil Engineering, Islamic Azad University of Mashhad, Iran.
Corresponding Email: (f.khamchin@gmail.com)

***Abstract* - The estimation of areal rainfall (daily, monthly, etc) is the basic need in meteorological projects. In this field, there are various methods, one of them is the finite element method. The present study aimed to estimate areal rainfall with a 16-year period (1997-2013) by using Galerkin's method (finite element) in Mashhad plain basin for 42 stations. Therefore, it was compared with other usual methods such as arithmetic mean, Thiessen, Kriging and IDW. The analysis of Thiessen, Kriging and IDW were in ArcGIS10.0 software and finite element analysis was done by using Matlab7.08 software. Isohyetal method was the base of comparison. The results showed that finite element had higher** accuracy **than arithmetic mean. There was almost the same precision in comparison with Kriging and IDW methods, while Thiessen had slight errors.**

***Key words* - Galerkins' method, Mashhad plain, Interpolation function, Areal rainfall**

1. INTRODUCTION

The hydrological models are very important tools for planning and management of water resources. These models can be used for identifying basin and nature problems and choosing various managements. Precipitation is based on these models. Calculations of rainfall would be affected by displacement and region factor such as topography, etc. Estimating areal rainfall is one of the basic needs in meteorological, water resources and others studies. There are various methods for the estimation of rainfall, which can be evaluated by using statistical data and mathematical terms. In hydrological analysis, areal rainfall is so important because of displacement of precipitation. Estimating areal rainfall is divided to three methods: 1- graphical. 2-topographical. 3-numerical [6].

This paper represented calculating mean precipitation (daily, monthly and annual) using Galerkin's method (numerical method) and it was compared with other methods such as kriging, IDW, Thiessen and arithmetic mean. In this study, there were 42 actual gauges (Table1) and thirteen dummies in Mashhad plain basin which is calculated by Galerkin's method. The method included the use of interpolation functions, allowing an accurate representation of shape and relief of catchment with numerical integration performed by Gaussian quadrature and represented the allocation of weights to stations [11].

Akin (1971) introduced the method of finite element analysis. The procedure for calculating areal rainfall, based on finite element methods, is presented by P. Hutchison (1972). This method was studied on Dunedin, New Zealand which consisted of eleven permanent and temporary gauges. Each rain gauge was allotted two weights, one associated with the rainfall reduced to datum, and the other with the rainfall-altitude relationship. The latter weight effectively removed any systematic errors due to altitudinal bias of the network. The rainfall- altitude relationship, derived from individual storms and synoptic situation for a small area, was used to show those errors due to the bias of the network can be considered.

2. MATERIAL AND METHODOLOGY

Mashhad plain basin is one of the thirteen Ghareghom sub basin. The total area is equal to 9909.9 km^2 which involves plain (3351 km^2) and mountain (6558 km^2) (Yasury,….). Mashhad plain is one of the important plain in Khorasan Razavi. Mashhad plain is located in longitude 58º 29′ to 59º 56′ east and latitude 35º 58′ to 37º 3′ north. In this paper there was 42 actual gauges and 13 dummy gauges within a period of 16-years (1997-2013).

Galerkin's method

The finite element method is a numerical procedure for obtaining solutions to many of the problems encountered in engineering analysis. First, it utilizes discrete elements to obtain the joint displacements and member forces of a structural framework and estimate areal precipitation. Second, it uses the continuum elements to obtain approximate solutions to heat transfer, fluid mechanics, and solid mechanics problems [13].

Galerkin's method is used to develop the finite element equations for the field problems. It uses the same functions for $N_i(x)$ that was used in the approximating equations. This approach is the basis of finite element method for problems involving first-derivative terms. This method yields the same result as the variational

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method when applied to differential equations that are self-adjoints.

Galerkin's method is almost simple and eliminates bias by representing the relief by suitable mathematical model and incorporating this into the integration [13]. In this paper, two powerful techniques were introduced which was applied in Galerkin's method:

(1) The use of interpolation functions to transform the shape of the element to a perfect square.
(2) The use of Gaussian quadrature to calculate rainfall depth numerically [11].

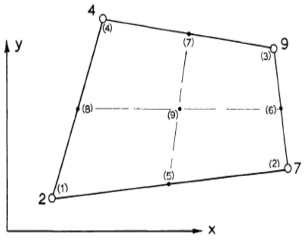

Fig.1-Global Coordinate System

In this study, Mashhad plain is divided to 40 elements which are quadrilateral. In each element, the rain gauge (Table.1) was situated on the node of the stations. The coordinates are given according to UTM, where x and y are the horizontal and z, the vertical (altitude) coordinate. It was necessary at the outset to number the corner nodes in a set manner and for the purpose of this paper, an anticlockwise convention was adopted.

TABLE 1
Specifications of Mashhad Plain

Station	x_utm	y_utm	z	station	x_utm	y_utm	z
Abghandferizi	685763	4044656	1380	Zoshk	697502	4024363	1880
Al	738067	4067314	1475	Sagh Bik	626453	4064663	1510
Mashhad office	731039	4021956	990	Torogh dam	729639	4006280	1240
Ardak	713617	4067555	1310	Kardeh dam	738455	4056330	1300
Olangasadi	752266	4015822	900	Shandiz's Sarasiyab	709820	4031347	1270
Androkh	738113	4051588	1200	Sharif Abad	725852	3989818	1455
Balghor	731891	4081022	1920	Esh Abad	664428	4018975	1346
Bahmajan Oliya	675941	4086234	1340	Ferizi	676802	4039812	1640
Tabarok Abad	652515	4117177	1510	Ghadir Abad	676396	4074984	1175
Talghor	710308	4078053	1540	Gharehtikan	784601	4079834	520
Jaghargh	708629	4021113	1420	Ghochan	633351.7	4103320.3	1350
Jong	731149	4073827	1700	Kabkan	669345	4124444	1435
Chakaneh Oliya	631555	4078712	1780	Golmakan	693844	4040097	1400
Chenaran	689618	4057478	1170	Golmakan[1]	704544.3	4039983.5	1176
Chahchaheh	797692	4060098	479	Gosh bala	728529	4066718	1580
Hesar(Kashafrod)	715841	4020953	1220	Mareshk	727140	4077931	1870
Darband	742634	4098004	970	Marosk	638479	4043758	1495
Derakht Tot	734269	3997037	1270	Mashhad	736569.7	4016743.5	999
Dolat Abad	694409	4035379	1510	Moghan	714164	4001945	1780
Dahane Akhlamad	674033	4051676	1460	Miyami	780656	4015387	1030
				Hendel Abad	768676	4035400	1210

[1] Meteorological station

It was assumed that rainfall (q) at a point (x,y) was a function of the altitude of the point then[11]:

$$q = r + \alpha z \qquad (1)$$

Where:
r: the rainfall at (x,y) reduced to the altitude datum;
α: ($\alpha = \beta r + \gamma$) the coefficient;
z: the altitude above a defined datum;
Within the element, it was assumed that at any point (x,y) the reduced rainfall, r can be represented as:

$$r = N_1 r_1 + N_2 r_2 + N_3 r_3 + N_4 r_4 \qquad (2)$$

N_1 to N_4 were weights corresponding to corner of quadrilaterals (nodes) and r_1 to r_4 were the reduced rainfall at the nodes.
In matrix notation equation (2) became:

$$r = [N]\{r_e\} \qquad (3)$$

Interpolation functions transformed the coordinates to local system (fig.2), it was written:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

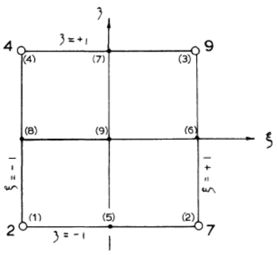

Fig.2-Local Coordinate System

If the altitude was considered, it was similar to reduced rainfall:

$$z = M_1 z_1 + M_2 z_2 + \ldots + M_4 z_4 \qquad (4)$$
$$z = [M]\{z_e\}$$ or

or The use of interpolation functions eliminated altitudinal bias from the network, since the number of intermediate points could be chosen to ensure that the ground surface was adequately represented.
The volume of precipitation on each element by integrating the rainfall over the area of the element was:

$$v = \int q\, dA = \iint_?^? q\,dx\,dy \qquad (5)$$

In general, it was difficult to integrate with respect to (x,y).

So it was transformed to local coordinate system.

$$v = \iint_{-1}^{+1} q\,|J|\,d\xi\, d\eta \qquad (6)$$

[J] : the Jacobian matrix which could be easily evaluated by numerical process[16]:

$$|J| = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \qquad (7)$$

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$$
$$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 \qquad (8)$$

$$[x\ y] = [N_1\ N_2\ N_3\ N_4] \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} \qquad (9)$$

Differentiating both sides with respect to ξ and η gives:

$$\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix}$$

$$\text{or } J = \begin{bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{bmatrix} [x_e\ y_e] \qquad (10)$$

since r, z and J could be expressed as functions of ξ and η equation (6) may be evaluated exactly using Gaussian quadrature, thus,

$$\iint_{-1}^{+1} f(\xi\eta)\,d\xi\,d\eta = \sum_{i=1}^{n} \sum_{j=1}^{n} H_i H_j f(a_i a_j) \qquad (11)$$

3. Results and Table

This paper represented the estimation of mean precipitation (daily, monthly and annual) in Mashhad plain by Galerkin's method which was compared with arithmetic mean, Thiessen, kriging and IDW. The values of Galerkin's method by Matlab7.08 software and Thiessen, kriging and IDW by ArcGIS10.0 were calculated. The base of the comparison was isohyetal method, because it showed the relief and took into account the effect of rain gauges, therefore it could represent rainfall data and region condition completely. The most accurate method was isohyetal method in estimating mean precipitation.
Cross-validation was usually used to compare the accuracy of interpolation method. In this study, root mean square error (RMSE) was used as validation criteria.
Meanwhile, in the present study, the effects of altitude were neglected for two reasons. First, partial correlation coefficient of $\frac{rainfall}{altitude}$ gradients was weak and second, the storms data were not accessible.

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Figures 3 to 5 indicated RMSE of arithmetic mean, Thiessen, Kriging and IDW in scale of daily, monthly and annual which were compared with other methods.

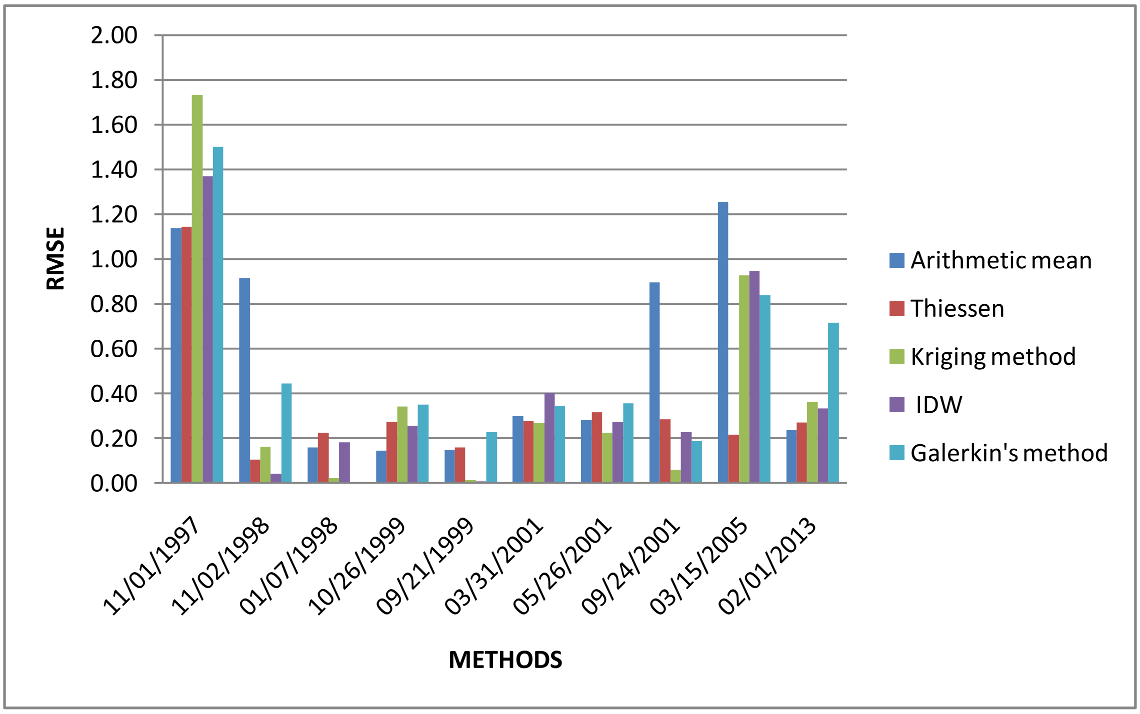

Fig.3- Comparison of daily precipitation

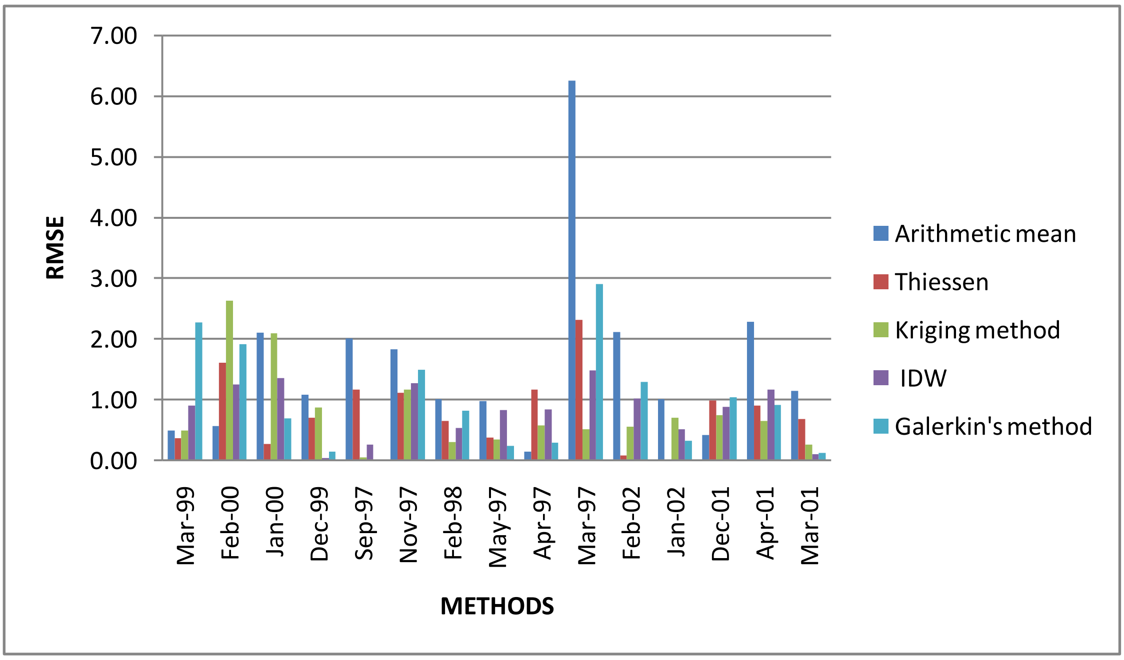

Fig.4- Comparison of monthly precipitation

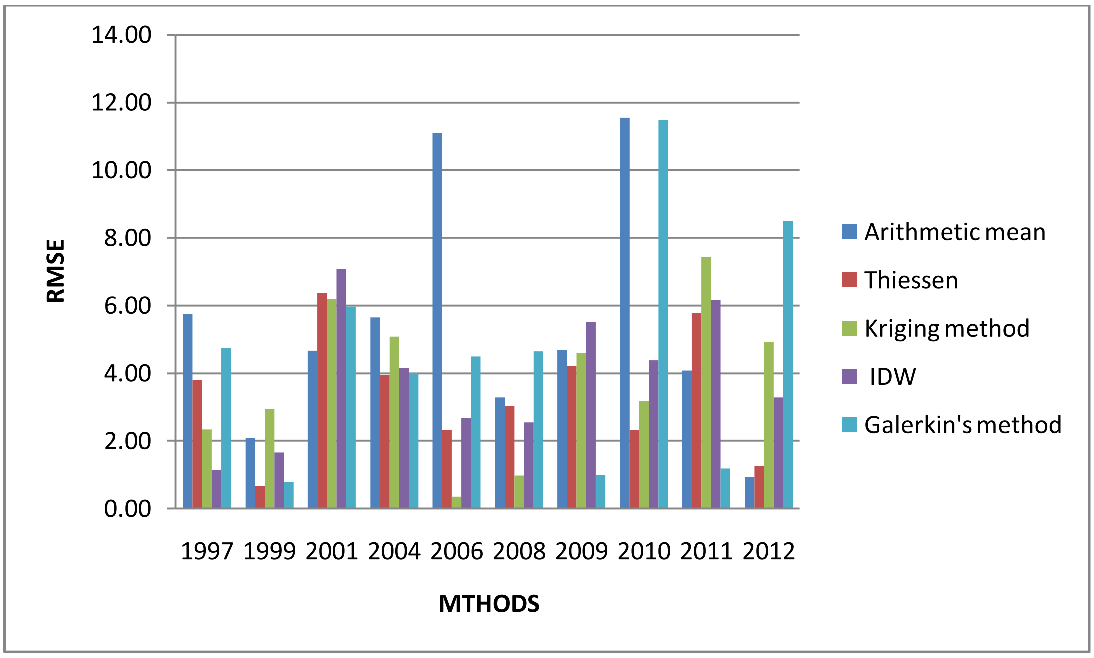

Fig.5- Comparison of annual precipitation

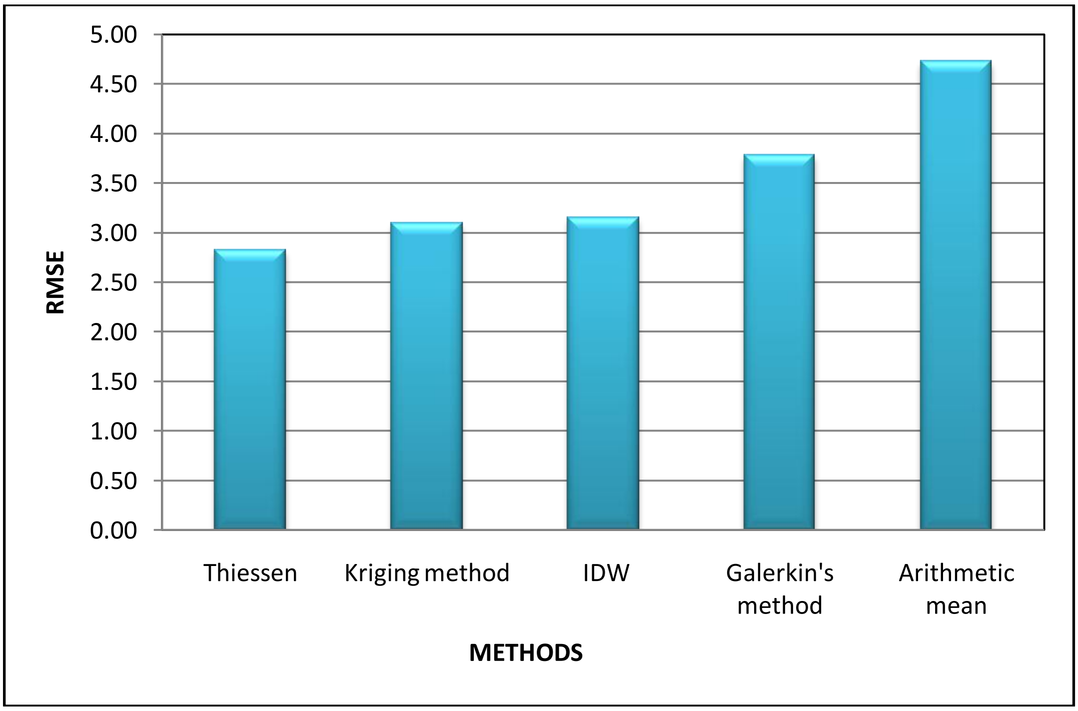

Fig.6- Comparison of methods

Meantime Kriging had a limitation, that is points of variogram moldel should be more than 50 points, otherwise, stochastic simulation is not reliable. In this study, 42 points were selected, so it could not strongly claim that the results were valid.

4. CONCLUSION

In this study, the estimation of areal rainfall by Galerkin's method was an innovative step. The case study was Mashhad basin (9909 km^2) which included 42 rain gauges. Comparing other methods indicated that:

1- Galerkin's method was more efficient in comparison with arithmetic mean and it had more accurate results.(Fig.6)

2- Result of Galerkin's method was similar to Kriging, IDW and Thiessen method. (Fig.6)

3- Unlike other methods, mesh of finite element (Fig.7) could be used for calculating runoff, sediment and temperature and it did not need station weights.

4- Even within one network the number of interpolation points can be varied, so that in a rugged region the number can be increased with little increase in effort, while in a more uniform region fewer are necessary [11].

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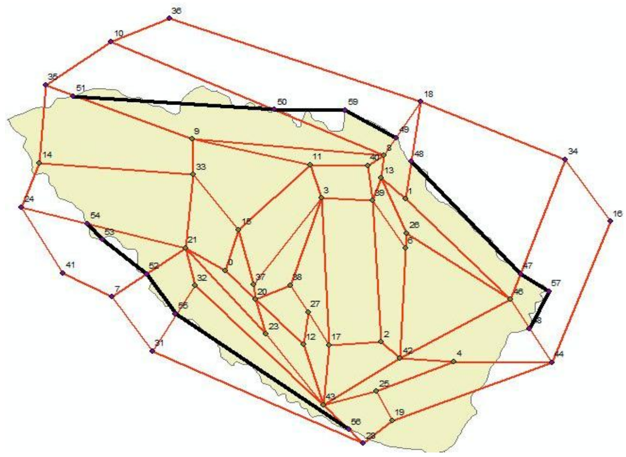

Fig.7- Mashhad Plain Basin network