

# Matched Filter Detectors for Next Generation Mobile Radio Channels

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**Abstract** — Matched filter is a linear filter which is designed to provide the maximum signal to noise power ratio at its output for a given transmitted symbol waveform. When the ISI is not present in the received signal, the detection based on the matched filter is an optimum detection process in the sense that it minimizes the probability of error at the front end of the receiver. This paper deals with the high speed data transmission techniques for new wireless and mobile systems. In particular, it looks at some of the signal processing techniques that are to be carried out in the receiver of such systems, because the mobile radio channels are characterized by frequency selective fast fading. Thus, the signal transmission performance is severely degraded because of such factors. Specifically, this paper deals with the detection using a matched filter followed by near maximum likelihood detector (NMLD) for the application of mobile radio environments. NMLD and its complexity depend on length and the numbers of stored vector. So, it is important to choose optimum NMLD, here we choose in this research four stored vectors while the length of the stored vector is of eight symbols length which is sometimes called as delay in detection. The performance of the detector is plotted in terms of bit error rate (BER) versus signal to noise ratio (SNR). Various cases of the matched filter based detector are studied in this paper. All the results obtained by this work are based on the channel with four independent fading paths. The channels used in this paper are generated and tested successfully before using them for the study of the detector. Channel used for this study is generated for the outdoor vehicular environment with the Doppler spread 100 Hz and with the delay spread nearly 14 microseconds.

**Keywords** — Matched Filters, Radio Physics, Mobile Radio, NML Detectors

## 1. INTRODUCTION

Matched filtering is a process known to be the optimum linear filtering for detecting a known signal in presence of random noise. In the matched filter, sets of consecutive samples from incoming signal are kept in memory of the receiver. At each matched-filter iteration stage, these samples are multiplied element wise to samples of known replica, and integrated to calculate

correlation between the memorized set of input samples and local replica [1-3]. It is well known [4-8] that the optimum filter for a received signal with no ISI and additive white Gaussian noise (AWGN) is one matched to the signal.

In its basic form, the matched filter provides output signal, which is the maximum likelihood estimate of the signal sample. After the processing of the received signal with matched filter, a simple threshold detector can be used for the detection at the output of the matched filter.

Unfortunately, in case of practical wireless channels, ISI is present in the received signal. Thus, the matched filter detector is then no longer optimum essentially because the ISI creates unwanted data symbols at the output of the matched filter.

Matched filtering of a received signal with inter-symbol interference (ISI) will still maximize the signal to noise ratio (SNR) without altering the ISI, but will "color" the noise. Any further filtering at this point to reduce the ISI will beat the expense of the SNR. Matched filter in conjunction with the various detector, provides excellent performance for delay spread from zero to one symbol interval [9].

A matched filter followed by the equalizer is the optimum linear system for a received signal with some ISI [1-5,10]. However, for severe multipath channels, a non-linear technique such as decision feedback equalization (DFE) or MLSE may be required [11]. In this paper, a matched filter, which is followed by NMLD, is used for the detection purpose for the channels, which are severely affected, by ISI as well as phase.

The model of the data transmission system, which is based on 4-QAM modulation scheme, is given in the figure 1. The aim of this chapter is to study the performance of the matched filter based NML detector and it is assumed that the channel is known to the receiver i.e. detector has the perfect knowledge of the channel at every time instant. A number of cases of the detector with matched filter have been studied; let us say system A, B, C and D.

## 2. ADVANTAGE OF MATCHED FILTER BASED DETECTOR

For a channel with  $g+1$  fading paths, the received signal sample is given by

$$r_i = \sum_{h=0}^g s_{i-h} y_{i,h} + w_i \quad (1)$$

For understanding the importance of the matched filter, a four paths mobile radio channel is considered here where the received signal is given by

$$r_i = s_{i-0} y_{i,0} + s_{i-1} y_{i,1} + s_{i-2} y_{i,2} + s_{i-3} y_{i,3} + w_i \quad (2)$$

Assuming that the receiver is trying to detect  $s_i$  with the help of received signal sample  $r_i$  at time instant  $t = iT$ .

Let us consider the case of the channel with impulse response:

$$Y_i = [0 \ 0 \ 0 \ 1] \quad (3)$$

In this case of the channel, the signal at the output of the channel is given by

$$r_i = s_{i-3} + w_i \quad (4)$$

i.e. the received signal coming at the front end of the receiver will not have the information about the transmitted symbols  $[s_i, s_{i-1}, s_{i-2}]$ . This will lead to a problem, as normally  $r_i$  is used to detect the symbol  $s_i$ .

The problem can however be circumvented by a matched filter. A full-length matched filter detector uses the received signal samples  $r_i, r_{i+1}, r_{i+2}, r_{i+3}$  for the detection of  $s_i$ . The detection process starts after arrival of the received signal sample  $r_{i+3}$ . Thus, even if there is no information of the symbols  $[s_i, s_{i-1}, s_{i-2}]$  in  $r_i$ , the matched filter based detector would still work using  $r_{i+3}$  as it would contain the desired symbol  $s_i$ , as demonstrated by the following equations 5-8.

$$r_i = s_{i-3} + w_i \quad (5)$$

$$r_{i+1} = s_{i-2} + w_{i+1} \quad (6)$$

$$r_{i+2} = s_{i-1} + w_{i+2} \quad (7)$$

$$r_{i+3} = s_i + w_{i+3} \quad (8)$$

Thus, it can be seen that how the matched filter based detector is smart enough to detect the desired data symbols.

### 2.1 System A: Full length matched filter based detector

This system is the full-length matched filter based NML Detector; the number of received signal samples used for the detection in this system depends on the number of independent fading paths in the channel. Thus for a detection of the symbol which has come at the front end of the receiver after passing from the wireless channel with  $g$  independent fading paths, matched filter requires  $g+1$  received signal samples for the correct detection in case of full length matched filter. Thus, for the case of a channel with four independent fading paths, the detector

uses four consecutive received signal samples to form a full-length matched filter to perform the detection at any time instant. Similarly, for the case of the channels with two and three paths independent fading paths, the matched filter based NML detector needs two and three consecutive received signal samples for the correct detection of the symbol.

To understand how the algorithm is working, let us considering the case of the channel with four independent fading paths. For the detection of  $s_i$ , the received signal samples  $r_i, r_{i+1}, \dots, r_{i+3}$  are taken into consideration. Using the general equation of the received signal sample is given by

$$r_i = \sum_{h=0}^g s_{i-h} y_{i,h} + w_i \quad (9)$$

the four consecutive received signal samples are given as:

$$r_i = s_i y_{i,0} + s_{i-1} y_{i,1} + s_{i-2} y_{i,2} + s_{i-3} y_{i,3} + w_i \quad (10)$$

$$r_{i+1} = s_{i+1} y_{i+1,0} + s_i y_{i+1,1} + s_{i-1} y_{i+1,2} + s_{i-2} y_{i+1,3} + w_{i+1} \quad (11)$$

$$r_{i+2} = s_{i+2} y_{i+2,0} + s_{i+1} y_{i+2,1} + s_i y_{i+2,2} + s_{i-1} y_{i+2,3} + w_{i+2} \quad (12)$$

$$r_{i+3} = s_{i+3} y_{i+3,0} + s_{i+2} y_{i+3,1} + s_{i+1} y_{i+3,2} + s_i y_{i+3,3} + w_{i+3} \quad (13)$$

Just prior, to receipt of signal sample  $r_{i+3}$ , the detector holds in store  $k$ -different  $n$ -component vectors in together with the cost as given in [9.12]. The vector  $R_i$  is formed which is having the four components as given by the equation 8.14.

$$R_i = [r_{i,0} \ r_{i,1} \ r_{i,2} \ r_{i,3}] \quad (14)$$

The channels impulse response for a four paths wireless channel is given by

$$Y_i = [y_{i,0} \ y_{i,1} \ y_{i,2} \ y_{i,3}] \quad (15)$$

If worst condition of the channel is considered then the channel impulse response may be given as

$$Y_i = [0 \ 0 \ 0 \ 1] \quad (16)$$

Then from equation 10,

$$r_i = s_{i-3} + w_i \quad (17)$$

Thus, the above equation has no information about  $s_i, s_{i-1}, s_{i-2}$ . This type of problem can be resolved with the help of matched filter by considering four consecutive received signal samples.

These four components  $r_{i,0}, r_{i,1}, r_{i,2}$  and  $r_{i,3}$  as given in equation 14 are derived from the equations 10-13 as given below with the all the terms containing already

detected symbols  $s'_{i-1}$ ,  $s'_{i-2}$  and  $s'_{i-3}$  removed. Thus,

$r_{i,0}$ ,  $r_{i,1}$ ,  $r_{i,2}$  and  $r_{i,3}$  are given by

$$r_{i,0} = r_i - (x_{i-1} y_{i,1} + x_{i-2} y_{i,2} + x_{i-3} y_{i,3}) \quad (18)$$

$$r_{i,1} = r_{i+1} - (x_{i-1} y_{i+1,2} + x_{i-2} y_{i+1,3}) \quad (19)$$

$$r_{i,2} = r_{i+2} - (x_{i-1} y_{i+2,3}) \quad (20)$$

$$r_{i,3} = r_{i+3} \quad (21)$$

If the earlier decisions made by the detector are correct, then  $\{x_{i-h}\}$  are same as  $\{s_{i-h}\}$ . Thus, the above equations can be modified as given below:

$$r_{i,0} = s_i y_{i,0} + w_i \quad (22)$$

$$r_{i,1} = s_{i+1} y_{i+1,0} + s_i y_{i+1,1} + w_{i+1} \quad (23)$$

$$r_{i,2} = s_{i+2} y_{i+2,0} + s_{i+1} y_{i+2,1} + s_i y_{i+2,2} + w_{i+2} \quad (24)$$

$$r_{i,3} = s_{i+3} y_{i+3,0} + s_{i+2} y_{i+3,1} + s_{i+1} y_{i+3,2} + s_{i+3} y_{i+3,3} + w_{i+3} \quad (25)$$

If  $Z_i$  is given by

$$Z_i = [y_{i,0} \ y_{i+1,1} \ y_{i+2,2} \ y_{i+3,3}] \quad (26)$$

$$Z_i^* = \begin{bmatrix} y_{i,0}^* \\ y_{i+1,1}^* \\ y_{i+2,2}^* \\ y_{i+3,3}^* \end{bmatrix} \quad (27)$$

then the signal  $R_i Z_i^*$  is formed as given below

$$R_i Z_i^* = \begin{bmatrix} r_{i,0} & r_{i,1} & r_{i,2} & r_{i,3} \end{bmatrix} \begin{bmatrix} y_{i,0}^* \\ y_{i+1,1}^* \\ y_{i+2,2}^* \\ y_{i+3,3}^* \end{bmatrix} \quad (28)$$

$$\begin{aligned} R_i Z_i^* &= (s_i y_{i,0} + w_i) y_{i,0}^* \\ &+ (s_{i+1} y_{i+1,0} + s_i y_{i+1,1} + w_{i+1}) y_{i+1,1}^* \\ &+ (s_{i+2} y_{i+2,0} + s_{i+1} y_{i+2,1} + s_i y_{i+2,2} + w_{i+2}) y_{i+2,2}^* \\ &+ (s_{i+3} y_{i+3,0} + s_{i+2} y_{i+3,1} + s_{i+1} y_{i+3,2} + s_i y_{i+3,3} + w_{i+3}) y_{i+3,3}^* \end{aligned} \quad (29)$$

This equation can be simplified as given below.

$$\begin{aligned} R_i Z_i^* &= s_i (y_{i,0} y_{i,0}^* + y_{i+1,1} y_{i+1,1}^* + y_{i+2,2} y_{i+2,2}^* + y_{i+3,3} y_{i+3,3}^*) \\ &+ s_{i+1} (y_{i+1,0} y_{i+1,1}^* + y_{i+2,1} y_{i+2,2}^* + y_{i+3,2} y_{i+3,3}^*) \\ &+ s_{i+2} (y_{i+2,0} y_{i+2,2}^* + y_{i+3,0} y_{i+3,3}^*) \\ &+ s_{i+3} (y_{i+3,0} y_{i+3,3}^*) \\ &+ (w_i y_{i,0}^* + w_{i+1} y_{i+1,1}^* + w_{i+2} y_{i+2,2}^* + w_{i+3} y_{i+3,3}^*) \end{aligned} \quad (30)$$

$$R_i Z_i^* = s_i |Z_i|^2 + \text{ISI term} + \text{Noise Term} \quad (31)$$

In the above equation,  $R_i Z_i^*$  does not have unbiased estimate with  $s_i$ , therefore a parameter  $\lambda_i$  is then defined which is given below.

$$\lambda_i = \frac{R_i Z_i^*}{|Z_i|^2} \quad (32)$$

Substituting the value of  $R_i Z_i^*$  from equation 31 into 32, one gets

$$\lambda_i = s_i + \text{noise component} \quad (33)$$

Thus  $\lambda_i$  represents an unbiased estimate of  $s_i$ . Thus, for finding which particular value of  $x_i$  is closest to  $\lambda_i$ , a temporary cost value  $C_i$ , which is based on the matched filter, is calculated with the help of accumulated permanent cost value  $U_{i-1}$  and it is given by

$$C_i = U_{i-1} + \Delta_i \quad (34)$$

Here  $\Delta_i$  is the incremental cost, which is given by

$$\Delta_i = |\lambda_i - x_i|^2 \quad (35)$$

Now, from the set of  $k \times m$  expended  $\{P_i\}$ , the vector with the smallest cost value is selected [9,11,12]. The value of  $x_{i-n}$  with the smallest cost is the detected symbol  $s'_{i-n}$  of transmitted symbol  $s_{i-n}$ . Now, any vector in  $\{P_i\}$  whose first component is different from  $s'_{i-n}$  is then discarding by assigning it to a higher value of cost  $C_i$ . Now k-vectors are selected from the remaining set of vectors in  $\{P_i\}$  including that from which  $s_{i-n}$  was detected. The first component  $x_{i-n}$  of each of these selected vectors  $\{P_i\}$  is now omitted without changing their costs. Once the selection of k-vectors is complete, the permanent cost  $U_i$  of these selected are calculated using the equation 36 as given by

$$U_i = U_{i-1} + \left| r_i - \sum_{h=0}^3 x_{i-h} y_{i,h} \right|^2 \quad (36)$$

The smallest value of these costs is now subtracted from each of the k-cost in order to avoid an unacceptable increase in the value of the costs over a long transmitted message. This operation does not create the difference between the permanent costs. These k-selected vectors in  $Q_i$  are then stored along with the value their cost function  $\{U_i\}$ . Now the detector is ready for the next step of detecting next symbol i.e.  $s_{i-n+1}$  on the receipt of the symbol  $r_{i+4}$ . This process will continue until the detection of last symbol in a message.

### 2.2 System B: Matched filter based detector using three received signal samples

The Matched filter based NML detector studied in the previous section was with full-length matched filter. However, in system A uses an NML detector based on matched filter using three received signal samples only. The three consecutive receives signal samples for the detection of  $s_i$  are  $r_i$ ,  $r_{i+1}$  and  $r_{i+2}$  given by

$$r_i = s_i y_{i,0} + s_{i-1} y_{i,1} + s_{i-2} y_{i,2} + s_{i-3} y_{i,3} + w_i \quad (37)$$

$$r_{i+1} = s_{i+1} y_{i+1,0} + s_i y_{i+1,1} + s_{i-1} y_{i+1,2} + s_{i-2} y_{i+1,3} + w_{i+1} \quad (38)$$

$$r_{i+2} = s_{i+2} y_{i+2,0} + s_{i+1} y_{i+2,1} + s_i y_{i+2,2} + s_{i-1} y_{i+2,3} + w_{i+2} \quad (39)$$

### 2.3 System C: Matched filter based Detector using two received signal samples

For the given channel having four independent fading paths, this section presents a study of matched filter based detector that uses two received signal samples  $r_i$  and  $r_{i+1}$  for the detection of  $s_i$  as given below:

$$r_i = s_i y_{i,0} + s_{i-1} y_{i,1} + s_{i-2} y_{i,2} + s_{i-3} y_{i,3} + w_i \quad (40)$$

$$r_{i+1} = s_{i+1} y_{i+1,0} + s_i y_{i+1,1} + s_{i-1} y_{i+1,2} + s_{i-2} y_{i+1,3} + w_{i+1} \quad (41)$$

The remaining part of the algorithm is similar as given in system A.

### 2.4 System D: Matched filter based detector using two received signal samples by optimizing the value of cost function

Like systems C, the system D also uses a matched filter detector based on two received signal samples  $r_i$  and  $r_{i+1}$  given by equations 40 and 41. However, the cost calculations are now slightly modified. A scaling constant ( $\alpha$ ) is introduced in this system, which is used

to optimize the results obtained from the matched filter. Thus, the incremental cost is given by:

$$\Delta_i = \alpha |\lambda_i - x_i|^2 \quad (42)$$

In the equation 42,  $\alpha$  is the scaling constant, which is used to optimize the performance of the detector. The performance based on this constant is given in figure 5. Now, from the set of  $k \times m$  expended  $\{P_i\}$ , the vector with the smallest cost value is selected. The value of  $x_{i-n}$  with the smallest cost is the detected symbol  $s'_{i-n}$  of transmitted symbol  $s_{i-n}$ . Now, any vector in  $\{P_i\}$  whose first component is differs from  $s'_{i-n}$  is then discarded by assigning it to a higher value of cost  $C_i$ . Now k vectors are selected from the remaining set of vectors in  $\{P_i\}$  including that from which  $s_{i-n}$  was detected. The first component  $x_{i-n}$  of each of these selected vectors  $\{P_i\}$  is now omitted without changing their costs. The remaining part is same as discussed in the previous systems.

## 3. COMPUTER SIMULATION RESULTS

The main objective is to plot the BER versus SNR curve for various schemes of the matched filter based detector. Like in other chapters one important assumption is also considered which that the detector has the perfect knowledge of the channel i.e. the perfect channel estimation is considered. In order to facilitate the proper comparison of the various schemes of the detector, the same fading sequences have been employed throughout the simulation tests. BER and SER are measured at different SNR so that the performance of the detector is not influenced by the choice of the particular fading sequence. Finally, the BER versus SNR curves plotted for each schemes.

The detection schemes investigated in this chapter are based on the matched filter used in conjunction with NMLD. The matched filter based NMLD studied in this chapter uses k (number of stored vectors) =4 and n (length of stored vectors) =8. All the systems studied in this chapter have been simulated for the channel with four independent fading paths in outdoor vehicular environment. The channel used has equal power distribution in the four fading paths (viz. CHOV-4EQ) [9, 12].

If the performance of systems A and B (as shown in figures 2 and 3 respectively) are compared, it is found that there is only a little difference at low SNRs. However at high SNRs, although both systems give irreducible error rates; but the performance of system B is better at high SNRs compared to the system A.

If the performance of systems B and C (as shown in figures 3 and 4 respectively) are compared, it is found

that system B is marginally better at low SNRs. However, at high SNRs, system C gives a much-improved performance compared to the system B.

The above results can be attributed to the following facts, at high SNRs, the performance of reduced length matched filter detector (systems B and C) is better compared to that of full-length matched filter detector (system A) because of a smaller amount of unwanted ISI. On the other hand, at low SNRs, the performance of full-length matched filter detector (system A) is quite good. This is attributed to the maximum effectiveness of the full-length matched filter compared to the reduced length matched filter.

As pointed out previously, system D is a modified form of system C obtained after modification of cost calculations by introduction of a scaling constant  $\alpha$ . The results of performance of system D are shown in figure 5 for different values of  $\alpha$ . Out of the four values of  $\alpha$  tried in this work, it is found that the best results are obtained for  $\alpha = 0.01$ .

In view of the significant change in results observed for various value of  $\alpha$ , it may be concluded that this is a useful parameter, which helps in controlling the level of ISI through modified cost calculations of stored vectors.

#### 4. CONCLUSIONS

It may be noted here that the worst case ISI channels studied here are really those which are less likely to occur in practice. However, the idea here is that if the ISI mitigation methods work well for these worst case wireless channels, then they are likely to perform better in practical channels with lesser degree of ISI. The data rates of 144 kbps used in this work may possibly appear to be small for future and next generation wireless systems under study. But a lot of next generation and future systems are likely to use OFDM where an incoming serial bit stream is split up into multiple parallel bit streams each of which then travels over a separate carrier. The reduced signal element duration on any of these individual carriers would however be of the same order as that used in this work, which is also another motivation behind this work. The NMLD used in this work is very least complex and thus the NMLD with the conjunction of matched filter can be used for the current mobile/wireless applications.

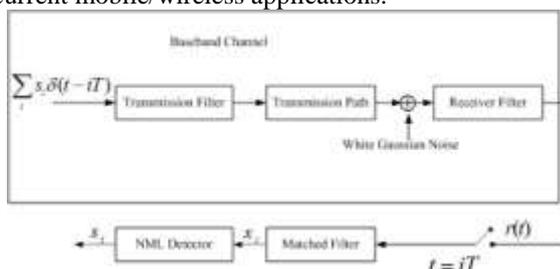


Fig. 1 Model of data transmission system with Matched filter

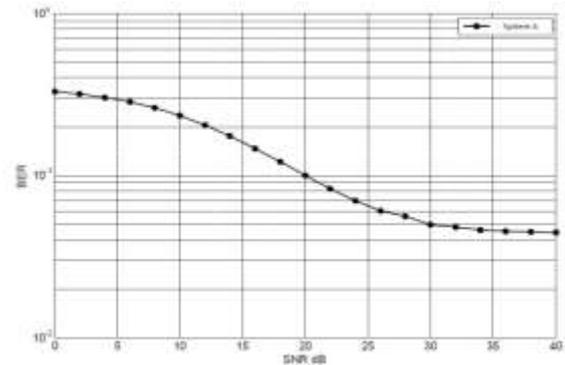


Fig. 2 Performance of matched filter based NML detector (system A)

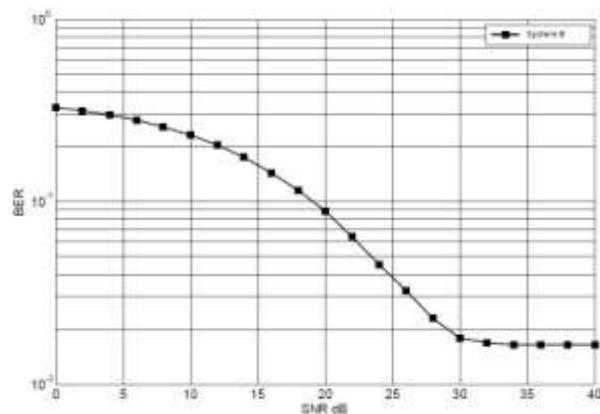


Fig. 3 Performance of matched filter based NML detector (system B)

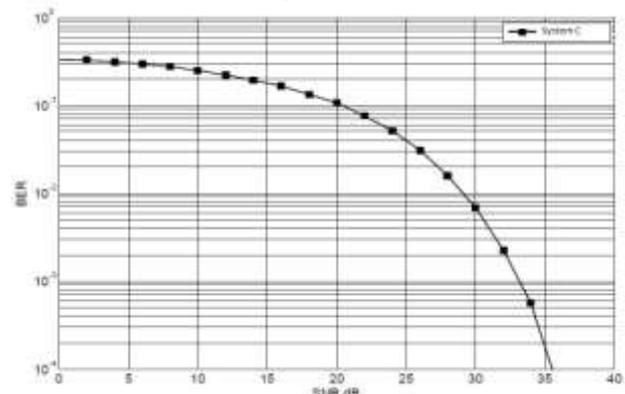


Fig. 4 Performance of matched filter based NML detector (System C)

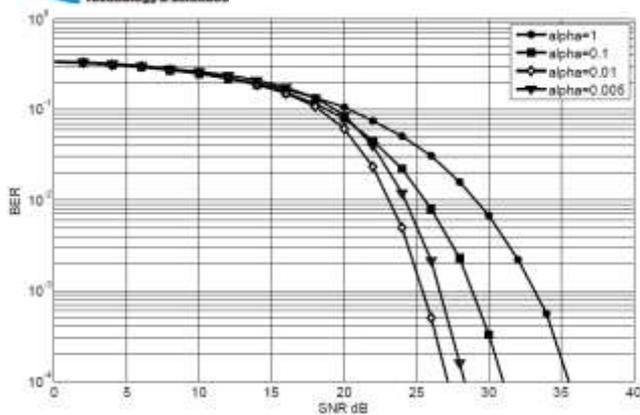


Fig. 5 Performance of matched filter based NML detector (System D)

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