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MAINTENANCE OF BUILDING AND PUBLIC WORKS INDUSTRY'S EQUIPMENT: SPARE PARTS INVENTORY MANAGEMENT'S COLLEGIALITY AS A CHALLENGE TO ECONOMIC PROFITABILITY

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ABSTRACT

Several aspects are highlighted in Building and Public Works' equipment management, among which, spare parts inventory management need to build up a spare parts inventory cannot be casual, as it is prescribed by the need to maintain maximum availability level under budget constraints and the storage policy's economic profitability as well. Several conventional inventory management policies were developed individually, not considering possible interactions, since construction companies' special characteristics is a simultaneous implementation of several projects but in different geographical areas distant from one another. This publication aims at studying the role and conditions of these various worksites' inventories' collegial management with the challenge of economic profit, obviously sought by any company. We clarify that New Information and Communication Technologies (NICT) are basic support for optimizing inventory management parameters like easy access to technical documentation, orders delivery time, inventories records, and inventory pooling. Finally, we highlight various construction companies' worksites' inventory pooling and orders pooling influence in the economic profit from spare parts inventory management policies.

KEYWORD – Inventory management policies, Spare parts, Public works equipment, Economic profitability.

INTRODUCTION

Inventory and spare parts supply management are in line with the overall issue in equipment management, especially modelling and use optimizing of construction equipment, very often exposed to random failures.

In any case, equipment management aims at its proper working when used; and breakdowns caused by some random events may affect this performance indicator (also called availability). These are so undesirable that they can have serious consequences, both human and financial. Yet, the operating equipment suffers from expected degradation, requiring permanent wear and spare parts (generally). Moreover, if the said parts required to replace faulty components are unavailable, the equipment persistently stops.

The spare parts are placed in storage to quickly replace faulty components and ensure business continuity to overcome these shortcomings. However, this stock building faces the clear constraint of these parts maintaining costs, which can significantly increase rental costs, namely the equipment's cost price. It then becomes sound to express the equipment seriously; components for which spare parts were stored within the limits of their economic return, knowing that, on the one hand, their shortage may be very costly to the company on the other, resources are not unlimited. The increasing competitiveness in the various construction projects means that managers are increasingly interested in spare parts inventory management, which is an important lever in the equipment maintenance costs optimizing policy.

In addition, physically disparate activities which require simultaneous management of equipment in geographically distinct areas particularly characterize most construction companies. This situation calls the challenge *of the collegial handling of several worksite spare parts inventories* through stores' virtual *pooling* for the company's overall economic profit.

1 INVENTORY MANAGEMENT GLOBAL PARAMETERS

We can keep the global parameters that affect storage cost:

- Purchase unit price
- Lead time represents the time between the moment the order is to be delivered and the order launching date
- An order's launching cost and cost of ownership: good inventory management aims to find the optimal number of launches, the launching cost of an order, and its cost of ownership to reduce the total launching cost over a one-year time horizon, for example.
- Inventory management policies: once we know the parameters involved in the storage cost calculation, we can determine inventory management policies that reduce the total storage cost.
- Inventory record and stock valuation

2 LITERATURE REVIEW: CONVENTIONAL INVENTORY MANAGEMENT POLICIES

Conventional inventory management policies are the first ones to be developed, namely since the 1930s. They ensure the management of the stock provided by supply systems, and the major aim is to satisfy the worksite's demand. Regularly, orders are placed for stocks replenishment. The time between the moment an order is placed and the reception of the parts is called *lead-time*, practically corresponding to deadlines caused by the order launching, the parts manufacturing (in some cases), and stockpiling.

In these policies, we are generally interested in two stock levels: *the net stock* (difference between physically available stock and not yet satisfied requests) and the stock position (including the net stock and already placed orders for which delivery is still expected). In the literature, the most commonly used conventional management policies are:

- The (s, Q) policy: "with continuous monitoring and reorder point".
- The (R, s) policy: "to replenishment period".
- The other versions are derived from the first two like the (s, S) policy, the (S-1, S) policy, the (R, s, S) policy, and the (R, s, Q) policy.

The **(s, Q) policy** is of continuous monitoring. It involves ordering a fixed quantity **Q** each time the stock position drops below a threshold called reorder point and noted **s**. The order is received at the end of the lead-time **τ**. Here, the time of ordering varies: if the order is bigger than average, the reorder point reached earlier; if the order slows down, the reorder point reached later. The correspondence between stock and order points aims to cover the demand until the order is received. Therefore, its level is equal to the demand during the lead-time (noted τ or LT). This policy is on continuous monitoring, *knowing anytime available stock must be alerted when an item reaches its reorder point*. It can practically cause high management costs (for example, establishing a computer-based monitoring system). Moreover, in case several products are from the same supplier, orders pooling cannot be done because all items do not necessarily reach their reorder points at the same time.

The **(R, S) policy** is also called the "periodical monitoring" or "periodical replenishment" policy. Each **R** period starting, if the position of the stock drops below a given value, called the replenishment level and noted S, a replenishment order is launched to bring the stock's position back to **S.**

Compared to the (s, Q) policy, the advantage of this policy is that it permits orders pooling per supplier, reducing shipping and ordering costs. According to [1], this is the most widely used periodic inspection policy; store staff can understand and operate it. Besides, the calculations are less complex than with the other periodic inspection models.

However, this policy has some *drawbacks*. *It is "blind" within a review period, so instantaneous variation in the demand keeps the system insensitive (unlike the (s, Q) policy, which is more reactive because of its continuous monitoring)*. In some cases, replenishment is carried out in small quantities, meaning that each period, if the stock level drops even slightly below S, an order must be placed to reach S if the relevant quantity is very small. That is why it is not recommended if the order's cost is high, as it would be better not systematically launch orders at each inspection time. In this case, the (s, S, R) version is more profitable as it suggests placing the order to bring the stock level back to S only if the stock level is less than or equal to s at the time of inspection.

Note: [2] is one of the first who studied the (s, Q) policies and (R, S) in the presence of a deterministic demand. After that, in the early fifties, those policies were developed for a stochastic demand case by [3], [4], [5], and [6].

The **(s, S) policy** is of continuous monitoring. As soon as the stock's position drops below the **s** order threshold, the stock position is replenished to a replenishment level **S.** Unlike the (s, Q) policy in which the ordered quantity is fixed, the order's size with this policy varies.

This model thus suggests reducing the stock level to **S** each time the net (physically available stock + units to receive, if any) stock level reaches the reorder point **s.**

[3] proved that there is an optimal solution to this inventory management system.

Consider a discrete-time inventory system in which one order is placed at each cycle, starting with deferred demands (orders are placed for quantities in shortage and received later). [7] establishes the expression of the cyclic average total cost CT (s, S) for this policy (s, S) from the renewal theory in the case where the cost structure and the parameters are static.

A simple enumeration algorithm can then obtain the optimal values s* and S*. Several algorithms were developed to reduce solutions' research area - The algorithm proposed by [8] and known as one of the most efficient [9].

[10] suggested another algorithm, which is supposed to reduce by 30% on average the number of required iterations by the Zheng and Federgruen algorithm.

The **(S-1, S) policy**, also known as "base rock", is very useful in basic spare parts (class A items) inventory management. With this policy, a quantity **S** of items is kept in stock. Each time an item is consumed, a unit order is placed to the next level to bring the stock level back from **S-1** to **S**: *this is a special case of the (s, S) system with s = S- 1.*

We also notice that when Q = 1, the system (s, Q) becomes equivalent to (S-1, S).

Several authors, including [11], [12], and [13], dealt with the question of determining the level of the optimal stock **S** for various cases.

Consider a stock with a maximum stock level **S**: independent and random unit requests randomly arrive at **λ** rate per time unit. Each request results in the release of a spare part and the order of a replacement part. The delivery time for this replacement part follows any distribution of **τ** average. If the stock is exhausted before delivering the spare parts, the **L** penalty is incurred for each request to meet by an emergency order delivered. A unit storage cost h per time unit is incurred for each stock unit.

The **(R, s, S) policy** periodically monitored system, in which both (s, Q) and (R, S) policies are combined. In fact, at the end of each monitoring period R, the position of the stock is reviewed. An order is only placed in this position is below an order's threshold noted **s.** The purpose of the ordered quantity is to bring the stock position back to a replenishment level **S**.

Compared to the (R, S) policy, the benefit of this policy is that it avoids placing too small orders if the demand has been low during the period; we can note that the (s, S) policy is a special case of the (R, s, S) policy which corresponds to the case in which the monitoring period R tends towards zero. So, the (R, s, S) policy may be considered a periodic version of the (s, S) policy. For the case where s = S-1, we also find the (R, S) policy.

The **(R, s, Q) policy** is also a period monitoring system, characterized by the combination of both policies (s, Q) and (R, S). We periodically review the state of the stock and each period R if the stock position is higher than an order threshold s, nothing is ordered. But if the stock position drops below the threshold s, we order a fixed quantity Q.

This policy is like the (R, s, S) policy, except that in this case, the supply is done through fixed quantities.

In addition, this system also permits avoiding placing too small orders if the demand during the period was very low, as is the case in the (R, S) policy. We notice that the (s, Q) policy represents a special case of the (R, s, Q) policy when the monitoring period tends towards zero (continuous monitoring). The WILSON model was instituted by Ford Harris in 1993 and is undoubtedly the most commonly used model. It is also known as the Economic Burst or Economic Quantity to Order (Q_{EC}) formula. This model is recommended when the demand rate and the replenishment time are known and constant, which may be the case for spare parts used only for systematic preventive maintenance. Preventive replacements' frequency is then fixed; the equipment manager, therefore, knows what time the replacements are expected and can supply the parts to receive them in due time to ensure the preventive replacements. Ironically, RH WILSON did not institute the said formula, but he just used this relationship in a management system that he commercialized, making it popular. The economic quantity to order Q^* and the optimal cycle duration T^* are given by

$$Q^* = \sqrt{\frac{2AD}{h}} \quad \text{and} \quad T^* = \sqrt{\frac{2A}{h \cdot D}}$$

[6] and [14] have shown that in terms of the total cost, the result of the economic quantity to be ordered is not very sensitive to parameters assessment errors. The partly explains the model's success and its widespread use in inventory management software. Note that there are several extensions of Wilson's model, but the most interesting is the one that deals with perishable foodstuffs, as it takes into account spare parts (staying in the store) degradation. If the decay rate **ε** is constant, then the stock's instantaneous level is given by [15].

<u>Note</u>

The conventional management systems were specified in this section for inventory management in general and did not include the maintenance aspect, which is spare parts' primary use. Maintenance actions integration into the spare parts supply and inventory management system thus proves essential in practice.

3 NICTs, AN IMPORTANT LEVER FOR PARTS COLLEGIAL MANAGEMENT

Global parameters for spare parts inventory management may be influenced by the emergence of New Information and Communication Technologies (NICT), in permitting now to reduce deadlines, ensure strict orders monitoring, access to suppliers around the world, benefit from profitable prices, discuss with suppliers and users of similar equipment, ensure a technological watch, and quickly access expertise in both technical and business fields.

Some construction equipment suppliers like Komatsu, Bomag, Hitachi, and Caterpillar currently guarantee two to four days' delivery times for critical parts worldwide. In addition, those equipment manufacturers present web portals that promote access to several millions of spare parts. The intelligent use of those technologies would thus undoubtedly contribute to improving the performance of spare parts inventory management systems [16].

This paper aims to study the effects of these stocks pooling on the various spare parts inventory-managing strategies through NICTs.
We thus pay attention to:

- Access to technical documentation
- Lead time reduction
- Inventory collective (collaborative) management.

3.1 Access to technical documentation

An increasing number of manufacturers are launching online websites where their clients have access to the technical documentation for their purchased equipment. Information regarding equipment updates is also displayed. Clients can download information to update the equipment or software packages. For example, Caterpillar, John Deere, and Hitachi (2020) provide their clients with parts catalogues, maintenance workers' training materials, diagnostic tools, and equipment performance data. Access to all this information from a simple workstation improves staff training, knowledge of the purchased equipment, and updates. It contributes to improving equipment reliability and availability [9]. Some manufacturers develop online discussion forums where equipment users can report problems and get answers from other users or the manufacturer's technical services. It is the case for the periodic distribution of service magazines where users' experience feedback and other innovations are brought to clients' attention (companies) as part of better after-sales service.

3.2 Reducing lead-time

One of the inventory management aspects the NICT (Internet especially) deeply impacted is lead-time. This period can be shortened thanks to online purchase transactions [17] and [18]. *Reducing the lead-time means reducing the stock security level likewise and consequently storage costs.*

For example, the purchasing manager fills out an order form he sends to the manufacturer or supplier in a traditional ordering procedure. Upon receiving the order form, the supplier manufactures and or prepares the requested batch and ships it to the requester. Now, with online catalogues, parts ordering procedures are simplified. It takes a series of mouse clicks on an image or drop-down menu to select the desired component accurately and avoid reference number transcription errors. After confirmation of the purchase, a cascade of logistical operations begins for the component delivery in a few hours (or days). The order to supplier transmission phase takes place quite instantly.

3.3 Collegial management of spare parts stocks (inter-worksites)

Geographically independent worksites' multiplicity characterizes construction companies. Most of those projects are managed independently, with various organizations. NICTs permit several inter-worksite collaboration ways. This e-collaboration between worksites is already implemented in various fields like trade, logistics, transport, the automotive and aeronautical industries [19] and [20]. But it must be systematized in Construction Companies.
E-collaboration can be *horizontal* when the projects (worksites) are at the same level of the supply chain, associated, especially with:

- Inventory pooling,
- Order pooling,

Vertical e-collaboration is when the construction company becomes a partner with its suppliers. It is the case, for example, with the supplier who manages his client's filtration or wear-parts stocks as part of the VMI (Vendor - Managed Inventory). Using interconnection technologies to profit in spare parts inventory management through information sharing and risk pooling is possible. Risks pooling, also called "statistical economies of scale", involves sharing risks between the various stakeholders or participants. The greater the number of participants, the least the individual effects (costs, impacts), This principle is the basis for insurance group schemes for property and person compensation high costs for losses incurred by a limited number of insured people are shared out to the total number of the insured, who then pay a reduced premium. This analogy to the insurance industry justifies translating the English term "risk pooling" by "mutualization des risques". In terms of inventory management, risks pooling can involve several initiatives such as

- Inventory pooling [21] and [22]
- Order pooling,
- Using similar or interchangeable spare parts (commonality / modularity) [24], [23] and [25].

3.3.1 Spare parts' inventory pooling.

Spare parts inventory pooling can be actual (physical) with several worksites supplied from only one centralized warehouse (see chart 1) or virtual [26] when each organization (worksite) keeps part of the stock on its worksite but can ship or receive parts from other worksites (see chart 2). In the latter case, we also speak of lateral transhipment [27] and [28]. Information sharing on stock levels becomes important and required for the system's good operation. Significant savings can be made if users of the same type of equipment decide to network their spare parts inventory management system [29] and [30]. Let us consider a set U of N worksites (projects) U_1, U_2,.., U_N which department in charge of the equipment decides to operate centralized inventory management. By disregarding parts' transfer costs between different sites, [21] showed that the total CT_I cost resulting from individual management is higher than the total CT_C cost resulting from collaborative management. If a spare component demand for the worksite U_i (i= 1, 2, .., N) follows a normal distribution averaging μ_i with a standard deviation σ_i. [21] shows that:

$$CT_C = K\sqrt{\sum_{i=1}^{N} \sigma_i^2} \qquad (1)$$

and $$CT_I = K\sum_{i=1}^{N} \sigma_i \qquad (2)$$

Therefore:

$CT_C \leq CT_I$ (K is a constant which depends on storage and shortage costs)

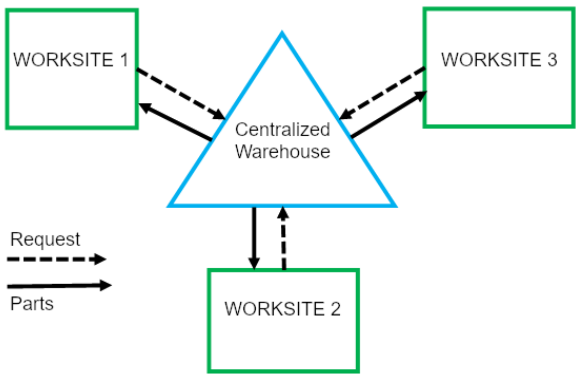

Chart1: Example of physical centralization

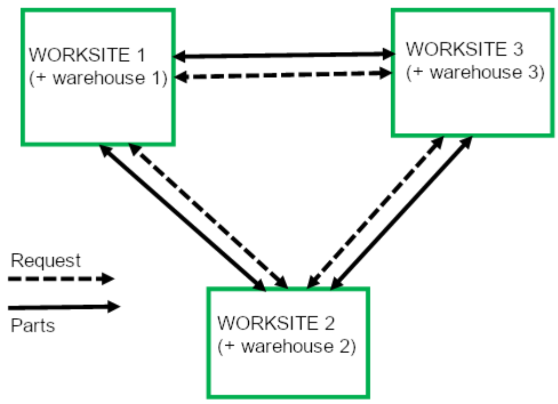

Chart2: Example of virtual centralization

Pooling parts is less expensive because lower demands on other worksites offset demands higher than the average appearing on one worksite. This saving is possible thanks to information sharing and transfer of parts possibility between worksites. [31] and [32] brought about extensions to Eppen's basic model results. [33] demonstrates that centralization remains good with maximizing a multi-installation system's profit. [34] bring changes to Eppen (1979) basic model by considering multi-stage systems and developing optimal ordering policies. [35] extend the results of the basic model by considering concave functions for storage and shortage costs. [36] prove that pooling is always profitable in a production system for stock in which a supplier serves several clients who use the (S-1, S) policy. Even though many research works show that stock pooling reduces stock levels, this should not be taken as a general rule since other works like those in [37] and [31] revealed, using counter-examples, that stock levels could increase after centralization. [36], [37], [38], [39], and [40] studied the conditions under which this "stock spooling defect" appears. [9] addresses the issue of determining the repairable spare parts quantity required to guarantee a given level of service (N_S). To ensure *(N_S)* service satisfaction level for a fleet of N machines with a failure rate $\boldsymbol{\lambda}$ handled by a repair workshop with ***c*** servers (repairers) each with a repair rate **μ**, the quantity ***y*** of spare parts to be kept in stock is given by the relation below [41]:

$$\sum_{i=0}^{y-1} Q_i \geq N_S \qquad (3)$$

Where

Q_i = i part default Probability
= Pr (i in the workshop | a breakdown is about to happen)

$$Q_i = \frac{N \cdot P_i}{N - \sum_{i=y}^{y+N}(i-y)P_i} \quad (0 \leq i \leq y) \qquad (4)$$

$$= \frac{(N-i+y) \cdot P_i}{N - \sum_{i=y}^{y+N}(i-y)P_i} \quad (y \leq i \leq y+N) \qquad (5)$$

P_i = probability, in a permanent state, that i out-of-order parts are waiting for a solution

$$= \frac{N}{i!} \quad ; 1 \leq i \leq c \text{ for } c \leq y \qquad (6)$$

$$= \frac{N^i}{c^{i-c}c!} \quad ; c \leq i \leq y \text{ for } c \leq y \qquad (7)$$

$$= \frac{N^y N!}{(N-i+y)!c^{i-c}c!} \quad ; y \leq i \leq y+N \text{ for } c \leq y \qquad (8)$$

$$= \frac{N^i}{i!} \quad ; 1 \leq i \leq y \text{ for } c > y \qquad (9)$$

$$= \frac{N^y N!}{(N-i+y)!i!} \quad ; y \leq i \leq c \text{ for } c > y \qquad (10)$$

$$= \frac{N^y N!}{(N-i+y)!c^{i-c}c!} \quad ; c \leq i \leq y+N \text{ for } c > y \qquad (11)$$

Knowing inevitably that $\sum_{i=0}^{y+N} P_i = 1$ is required, then we draw from it, P_0

$$P_0 = \left[1+\sum_{i=1}^{c-1}\frac{N}{i!}+\sum_{i=c}^{y}\frac{N^i}{c^{i-c}c!}+\sum_{i=y+1}^{y+N}\frac{N^y N!}{(N-i+y)!c^{i-c}c!}\left(\frac{\lambda}{\mu}\right)^i\right]^{-1} ;$$
for $c \leq y$ (12)

$$= \left[1+\sum_{i=1}^{y-1}\frac{N^i}{i!}\right]^{-1} \quad ; \text{for } c > y \qquad (13)$$

For $\mathbf{N_S}$, **λ**, □ and **c** parameters arbitrary values, by varying **N** (the number of machines in the fleet) and we get different ***y*** values of required spare parts. The results are registered in Table 1 (below). The curves in

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chart 3 show the evolution of the number of spare parts required per machine (**y / N**) according to the total number **N** of machines in the fleet.

Table 1: Number of Machines in the fleet

Number of machines in the fleet (N)	NS = 99%		NS = 95%	
	Number of PDR (y)	Number of PDR per machine (y/N)	Number of PDR (y)	Number of PDR per machine (y/N)
6	8,4	1,4	4,8	0,8
10	7	0,7	5	0,5
15	8,55	0,57	6	0,4
20	8	0,4	6	0,3
25	8,75	0,35	6,75	0,27
50	12,5	0,25	10	0,2

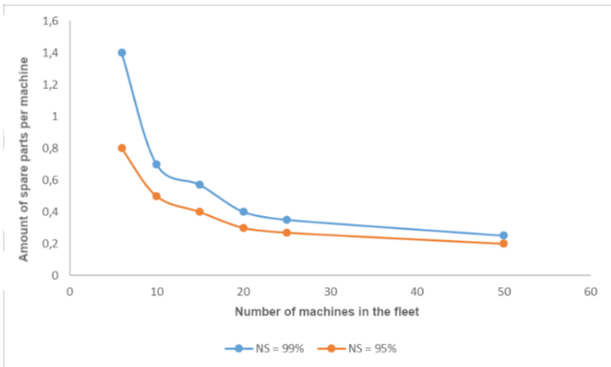

Chart 3: Change of the number of spare parts per machine depending on the size N of the machines fleet

For the same level of service, we noticed that this quantity of parts required per machine decreases when the total number of machines increases. Inventory pooling permits, in those cases, to reduce the quantity of spare parts.

The same result is obtained by [42] for airline spare parts stocks. However, it should be noted that mathematical models' heavy nature turn this result's analytical demonstration difficult. [36] demonstrate this pooling effect analytically by considering a production system for stock that [9] presents below by adapting it to the repairer's problem.

Consider the following problem:

A fleet of machines comprises N i.i.d machines (independent and identically distributed). Each **i** machine *has* a constant failure rate $\boldsymbol{\lambda_i}$ ($\lambda_i = \frac{\lambda}{N}$) and has *its* stock of $_i$ spare parts. Each time (**i**) the machine breaks down, a part is taken from the stock to replace the defective part if it is still furnished. The latter is sent to the workshop for repair. The repair workshop has a constant repair rate □. This system is equivalent to the one described by [36].

If the machines are identical, the quantities of spare parts ***s*** to be supplied for each machine to guarantee a level of service N_S ($N_S = 1 - \alpha$) are specified by [36]:

$$r^s \leq \alpha \qquad (14)$$

Where $r = \frac{\rho}{(N - N\rho + \rho)}$ with $\rho = \frac{\lambda}{\mu}$ ($\rho < 1$)

And $s = s_1 = s_2 = \ldots = s_i = s_N$

If a single spare parts stock is constituted for all the machines, then the system is equivalent to a single machine (N = 1) with a failure rate **λ** hence **r** = □, which is got by setting N = 1. To satisfy an *NS service level,* we, therefore, need a stock $\boldsymbol{s^c}$ such as:

$$\boldsymbol{\rho^{s^c} \leq \alpha} \qquad (15)$$

Stock centralization can only be profitable if and only if:

$$\boldsymbol{s^c} < \sum_{i=1}^{N} s_i$$

That is $s^c < N \cdot s$ (16)

For profitable centralization, if we use $s^+ = N \bullet s$ as the centralized stock maximum level, then we should easily meet the requirement:

$$\rho^{s^+} \leq \alpha \qquad (17)$$

Consider $\rho^{N \cdot s} \leq \alpha$

However, we already set that $[\frac{\rho}{(N - N\rho + \rho)}]^s \leq \alpha$

Then we should just show that

$$\rho^N \leq \frac{\rho}{(N - N\rho + \rho)}$$

i.e

$$N \cdot \rho^{N-1} - N \cdot \rho + \rho^N \leq 1 \ (\rho \neq 0)$$

$$N \cdot \rho^{N-1} \leq 1 - \rho^N + N \cdot \rho$$

$$N \cdot \rho^{N-1} \leq (1-\rho) \cdot \sum_{k=0}^{N-1} \rho^k + N \cdot \rho$$

$$\frac{N \cdot (\rho^{N-1} - \rho)}{1-\rho} \leq \sum_{k=0}^{N-1} \rho^k \qquad (18)$$

This inequality is always true since, for $\rho < 1$, the term on the left is always negative, and the one on the right is always positive. We can therefore state that for this configuration, pooling always permits reducing spare parts quantity, which is not the case for the following problem. Let us consider *N* worksites managing their respective stock of a non-repairable component according to the "base stock" policy (s -1, s). For each worksite, independent unit demands arrive randomly at a rate λ per time unit (Fish procedure). Each request results in the release of a spare part and the order for a replacement part. Each ordered part delivery time follows any average **τ** distribution. According to [9], the maximum quantity ***S*** of parts to keep, in an individualized management context, to ensure a level of service N_S ($N_S = 1 - \alpha$) is the first ***S*** value which proves the following relation:

$$\lambda \frac{\rho^S}{S!} \leq \alpha \qquad (19)$$

where $\rho = \frac{\lambda}{\mu}$ ($\mu = \frac{1}{\tau}$)

If the stocks of those *N* worksites are pooled, then the maximum quantity $\boldsymbol{S_c}$ of parts to keep, in a context of centralized management, to limit the risk of shortage to a predetermined threshold □, is the first value of $\boldsymbol{S_c}$ which testifies the relation:

$$N\lambda \frac{(N\rho)^{S_c}}{S_c!} \leq \alpha \qquad (20)$$

<u>Example</u> 1

For λ = 0.5; τ = 15; α = 0.05; N = 10,

We get S = 21 and S_c = 205, which is much below 210 (210 = 21 x 10). So there is a reduction in the stock level. This saving is noticed even though we consider actual (non-rounded) values of *S.*
However, this reduction is not always guaranteed, as shown in Example 2 below.

Example 2
For λ = 0.35; τ = 15; α = 0.05; N = 10,
We get S = 14 and $S_{c=}$ 144 which is superior to 140.
This example shows that the stock's level reduction is not always possible: this is what Yang and Schrage (2003) called "Inventory pooling anomaly ".

If the centralization of N stocks is profitable, then by using N times the individual optimal stock S as maximum stock for the centralized system, we must get a shortage probability $\tilde{\alpha}$ *at* the most equal to α, which is equivalent to $\frac{\tilde{\alpha}}{\alpha} \leq 1$

$$\tilde{\alpha} = N\lambda \cdot \frac{(N\rho)^{N\cdot S}}{(N\cdot S)!} \qquad (21)$$

Consider $G = \frac{\tilde{\alpha}}{\alpha} = \frac{N\lambda(N\rho)^{N\cdot S}}{(N\cdot S)!} \cdot \frac{S!}{\lambda\rho^S}$

After simplifications and use of Stirling's formula ($a! = a^{\frac{a+1}{2}} \cdot e^{-a} \cdot \sqrt{2\pi}$, we get)

$$G = [\frac{e\cdot N\cdot\rho}{S}]^S \cdot N^{\left(S-N\cdot S-\frac{1}{2}\right)} \qquad (22)$$

Inventory pooling reduces the level of stocks if ***G < 1***

For example, 1 above, the G value got is 3,54. In such a case, pooling does not reduce stocks. For example 2, we get G = 0,012. It is in line with the reduction in the observed inventory level. Even though inventory pooling is not synonymous with stock reduction, it is always proved to lower the total operation cost [36] and [31]. Studies on several industrial examples [23] assert that the profits from inventory pooling are generally smaller than those obtained through orders pooling.

3.3.2 Orders pooling

The pooling of orders from several worksites permits substantial economies of scale regarding purchase and transport costs [43] and [44]. Consider that **N** worksites decide to pool their orders for a given spare part from the same supplier. Because storage and shortage costs are not identical from one worksite to another and differences in transport costs and lead-time also exist, we can consider **N** products even though it is the same spare part. The orders' coordination problem for a product distributed over **N** facilities becomes equivalent to those for **N** products from only one facility. Therefore, the pooling of inter-worksite orders reduces to the classic problem of several items simultaneous management (Joint Replenishment Problem, JRP). One of the participating worksites, we call worksite 1, may be selected to place the order, receive the delivery, share it out into individual lots, and possibly ship to the other sites if the latter does not come to collect their items. The literature dealt with JRP. Thus, [45] present a review of *deterministic* and *stochastic* models. In practice, a warehouse has a variety of items to manage. Rather than supplying each item separately, savings can be made by coordinating same source items replenishments (same supplier or same geographic area). In so doing, order placement, handling, and transport costs can be shared over several products. The order cost comprises a major cost incurred with each order regardless of the ordered quantity and a minor cost incurred by each product included. In the inter-worksite orders pooling context, the major order cost is the sum of the expenses incurred by worksite **1,** which is responsible for placing the orders, while the minor order costs represent the costs of transport and transaction between worksite1 and each of the other worksites (see chart 4).

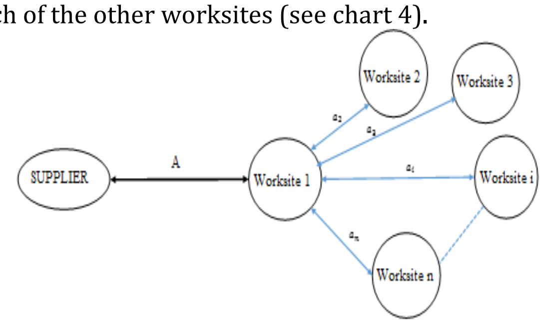

Chart 4: Order costs allocation between sites

Where A: major order placing cost
a_i: order minor cost if the item i is included in the order

The Orders' pooling drawback is that some items included in the lot are not ordered at their specific optimal cycle. As orders are placed in advance for most items, additional storage costs are incurred. Therefore, the problem is to find a compromise between the reduction in ordering costs and additional storage costs.
Note: the interchangeability of construction equipment spare parts is a maintaining factor very interesting in reducing the overall stock level and, therefore, the overall storage cost.

3.3.2.1 Deterministic models

In the literature, two methods are mainly proposed: *direct* pooling and *indirect* pooling. The direct pooling method divides the n items to order into *m* separated groups (m < n) and determines a fixed order cycle common to each group's items. Indirect pooling uses a basic fixed order cycle T*. It* determines for each item i (i = l, 2, ..., n) an ordering periodicity R_i which is a multiple of T*.* Solving the problem then consists in determining, for each item *i,* the positive integer k_i such as $R_i = k_i \cdot T$. Several comparative studies, including [46] and [47], grant a slight superiority to the indirect pooling method, which is also more widely covered in the literature. This principle is similar to opportunistic maintenance.

- The indirect pooling method

Consider:
D_i: demand rate for item i (units / time unit)
Q_i: quantity to order for an item (i units)

T: basic order or supply cycle (time unit)
R_i: order cycle of item i ($R_i = k_i \cdot T$), is a multiple of T
h_i: storage cost for item i (um / unit / time unit)
A: major order placing the cost
a_i: order minor cost if the item i is included in the order
CT: Total cost per time unit.

The expression of the total cost per time unit is the sum of the total order cost per time unit and the total storage cost per time unit: $CT(k_1, k_2,.., k_n, n) = \frac{1}{T}[A + \sum_{i=1}^{n}(\frac{a_i}{k_i})] + \frac{T}{2}\sum_{i=1}^{n} D_i \cdot h_i \cdot k_i$ (22)

Solving the equation $\frac{\partial CT}{\partial T} = 0$, gives the optimal value T^* of T for a given set of k_i.
The expression for T^* is given by:

$$T^* = [2\frac{(A+\sum_{i=1}^{n} a_i)}{\sum_i^n D_i h_i}]^{1/2} \quad (23)$$

The optimal total cost $CT^*(k_i)$ will then be:

$$CT^*(k_i) = [2 (A + \sum_{i=1}^{n}\left(\frac{a_i}{k_i}\right) \cdot (\sum_{i=1}^{n} k_i D_i h_i)^{1/2} \quad (24)$$

The next step is to determine the integers k_i (i = 0,1,… n) which minimize CT^*. For low values of n, the optimal solution is obtained with a simple enumeration procedure. However, the number of issues to explore explodes very quickly, and it becomes necessary to resort to heuristics. Apart from Goyal and Satir heuristics presented in a review in 1989 and permitting to reduce the research space, several others were proposed, including the algorithm of [48]. The latter results show an improvement compared to other methods available at the time of its publication. [49] develop a new research lower limit and improve [48] performances. The heuristics proposed by [50], [51], [52] and [53] permit getting solutions very close to the optimum.

- The direct pooling method

Very little research is devoted to this method, which divides the n items into m separated groups and finds an order cycle specific to each group.
Consider:
m: number of groups
j: group index, j = 1, 2, ..., m
G_j: the group number j
T_j: order cycle for the items of the group;

In the case of direct pooling, the total cost per time unit is expressed by:

$$CT = \sum_{j=1}^{m}[\frac{A+\sum_{i\in G_j} a_i}{T_j} + \frac{T_j}{2} \cdot \sum_{i\in G_j} D_i h_i] \quad (25)$$

By solving the equation $\frac{\partial CT}{\partial T} = 0$, we get the optimal value T^* of T_j:

$$T_j^* = [2 \cdot \frac{A+\sum_{i\in G_j} a_i}{\sum_{i\in G_j} D_i h_i}]^{1/2} \quad (26)$$

We then determine one set of m groups of one or more items, then calculate T^* for each group. For low values of *n,* it is easy to enumerate all possible combinations of *m* groups to determine the value m and the combination that gives the minimum total cost. For higher values of *n,* a heuristic like the one proposed by [54] is used to make the pooling. A comparative study of the direct and indirect methods carried out by [47] permitted finding the thresholds of the ratio $\frac{A}{a_i}$ depending on n from which the indirect method was outdoing the direct method. [55] used the technique of genetic algorithms to get solutions for order coordination issues.

3.3.2.2 Stochastic models

Several policies were proposed to address orders coordination concerns when the requests are done randomly. We can identify continuous review systems and periodic review systems. Until very recently, the can-order policy, we translate "commande opportuniste", proposed by [56], and the *QS* policy proposed by [57] and [58], was practically the only continuous review policies discussed in the literature. According to the can-order policy (s,c, S), each ***i*** item has three management parameters: *a maximum level of replenishment (S_i), a level from which can-order is permitted (c_i), and a mandatory order point (s_i) such that $s_i \leq c_i < S_i$.*
When the inventory of the ***i*** item reaches its mandatory order point s_i, the ordering process starts. Before placing the order, we check all other items j (j≠ i) stock levels. Those which inventory level is below or equal to their can-order are included in the order of ***i*** item by placing the order for required quantities to bring their inventory back to the replenishment level S_i. For the item that activates the order, this is a normal supply, while for the other items, it is referred to as a reduced cost order opportunity. [59] proved that can-order policies are not optimal for the problem. [60] established that it is not always possible to get an accurate optimal policy. Several heuristics are then proposed in the literature to calculate the parameters (s, c, S) (see [61], [62], [63], [64], [65] and [66]). The QS policy also noted ($Q, S_1, S_2, .., S_n$), operates as follows: each time the total consumption of the n items since the previous order reaches the quantity Q, an order is placed to bring each ***i*** item stock's level back to S. The tests carried out by [58] show that this policy gives good results when the number of items (and therefore companies) is low. The cost and demand parameters are similar. Upon recommendation of [67] and [44], develop a new continuous review policy Q (s, S) according to which total consumption is continuously monitored while for the item's inventories it is only periodically when total consumption reaches Q. Then each item's stock is reviewed according to the (s, S) policy. When $s_i = S_i - 1$, the Q(s, S) policy becomes the QS policy. Multi-product versions of the one-article periodical review systems were developed. The ***i*** Item inventory level is reviewed each period of T_i length. S_i replenishment is carried out if the stock level is below or equal to a given value. T_i frequencies are multiples from a T basic periodicity, as in the deterministic case with indirect pooling in section 3.3.2.1. Two heuristics for management parameters calculation were proposed by [68]. Those authors compared the results obtained by the models of policies (s, c, S) and those of periodic policies (S, T). They conclude that periodic policies give

the best results. However, we should note that these comparisons deal with used heuristics' efficiency in each policy and not with the policies themselves. For a long time, therefore, periodic policies' heuristics produced better results. However, the new heuristics developed for policies (s, c, S) by [65] and [69] give higher results than the results of periodic policies' heuristics when there are important fluctuations in demand. Another policy with the periodic review was proposed by [70] noted P (S, s). The inventory is reviewed at each time unit **t** interval. At each review, an (s, S) type policy is applied to each ***i*** item whose stock's level is below or equal to $\mathbf{S_i}$. A resolution procedure is applied to determine the management parameters **t**, $\mathbf{s_i}$, and $\mathbf{S_i}$, for each **t** value, find the optimal policy (s, S) for each item by the [71] algorithm. *The solution is the value of **t**, which gives the smallest total cost.* We just dealt with the profits drawn from inventory pooling in section 3.3.1 and from the orders pooling in section 3.3.2. Both initiatives involve worksites at the same level (grade) in the supply network. Section 3.3.2.3 deals with the example of coordination between two worksites located at different levels in the network, such as a worksite and its supplier.

3.3.2.3 The case of a worksite and its supplier

To illustrate the potential benefits of coordinating orders, consider a worksite and its supplier in a context where demand is known and stable.

Consider A: major order placing the cost
D: permanent demand rate
C_a: the unit supply cost from the supplier
S: Production-launching cost for each batch ordered.
P: the supplier's production rate (with P > D).
C_p: the unit production cost
ξ: the unit storage cost of the item per time unit per monetary unit

- Consider *the case where there is no coordination between supplier and worksite*. Each then determines its economic quantity, disregarding the others.

The economic quantities to produce and order, respectively,
Q_{EP} and Q_{EC} are given by:

$$Q_{EP} = \sqrt{\frac{2SP}{\xi C_p}} \qquad (28)$$

and

$$Q_{EC} = \sqrt{\frac{2AD}{\xi C_a}} \qquad (29)$$

The total cost expression in each case is given by:

$$CT\,(Q_{EP}) = S\cdot \frac{D}{Q_{EP}} + \xi \cdot C_p \cdot \frac{Q_{EP}}{2}\cdot\frac{D}{P} \qquad (30)$$

and

$$CT\,(Q_{EC}) = A\cdot\frac{D}{Q_{EC}} + \xi\cdot C_a\cdot\frac{Q_{EC}}{2} \qquad (31)$$

Either the worksite has enough power to require the supplier to deliver the accurate quantity Q_{EC} he needs, or the supplier has enough power and only sells Q_{EP} size batches.

Consider Q' (Q' = Q_{EC} or Q '= Q_{EP}) the quantity traded between the two.

The total *CTSC* cost incurred by both partners if there is no coordination is worth:

$$CTSC = CT\,(Q') + CT\,(Q') \qquad (32)$$

- Let us consider now *the case where the quantity to be manufactured for delivery to the worksite is determined in coordination (jointly) as suggested by* [72]. Each ordered batch is produced at one go and delivered. The total joint cost is given by [73]:

$$CTC\,(Q) = \frac{D}{Q}(S + A) + \frac{Q}{2}\cdot\xi\cdot(\frac{D}{P}C_p + C_a) \qquad (33)$$

Moreover, the optimal economic quantity to manufacture to meet each order is

$$Q^* = \sqrt{\frac{2D(S+A)}{\xi\left(C_p\frac{D}{P}+C_a\right)}} \qquad (34)$$

Author [73] Also, the supplier can anticipate the orders and manufacture a number ***n*** of batches to meet the current order and (n-1) subsequent orders. It shows that the total cost is then worth:

$$CTC\,(Q,\ n) = \frac{D}{Q}(\frac{S}{n} + A + \frac{Q}{2}\xi\left(C_a - C_p + nC_p\left(1+\frac{D}{P}\right)\right) \qquad (35)$$

The optimal value n^* must meet the following condition:

$$n^*(n^*-1) \le \frac{S(C_a-C_p)}{AC_p\left(1+\frac{D}{P}\right)} \le n^*(n^*+1) \qquad (36)$$

When n^* is known, the quantity to order is obtained by:

$$Q_{EC}\,(n^*) = [\frac{2D\left(\frac{S}{n^*}+A\right)}{\xi\left(C_a-C_p+n^*\cdot C_p\cdot\left(1+\frac{D}{P}\right)\right)}]^{1/2} \qquad (37)$$

and the quantity to manufacture is then $n^*Q(n^*)$.
The total costs incurred respectively by the worksite (CTC_c) and its supplier (CTC_f) are:

$$CTC_c = \frac{D\cdot A}{Q(n^*)} + \frac{Q(n^*)}{2} rC_a \qquad (38)$$

$$CTC_f = \frac{D\cdot S}{n^*\cdot Q(n^*)} + \frac{Q(n^*)}{2}\xi C_p\left[n^*\cdot\left(1+\frac{D}{P}\right)-1\right] \qquad (39)$$

Then the total costs incurred by both partners are:

$$CTC = CTC_c + CTC_f \qquad (40)$$

<u>Important note</u>:
Due to the construction companies' specific case, we finally face a multitude of virtually independent worksites (projects) which appear to each other as suppliers and clients. More practically, a worksite A can be both a "supplier" of a worksite B and a "client" of a worksite C in the spare parts' movement management.

<u>Numeric example</u>
Consider the example determined by the following parameters: D = 1800 units per year; P = 3500 units per year; A = 50,000 CFA F per order; S = 225,000 CFA F per launching; C_a= 25,000 CFA F per unit, C_p= 13,000 CFA F per unit and ξ = 0.15

- *Without* worksite-supplier coordination, we have:
  Q_{EC} = 219 units
  Q_{EP} = 917 units
  $CTSC\ (Q\ '= Q_{EC})$ = 2,776,450 CFA per year
  $CTSC\ (Q\ '= Q_{EP})$ = 2,701,300 CFA per year

- *With* worksite-supplier coordination, we have:
  n^* = 2
  $Q\ (n^*)$ = 278 units to be ordered each time by the supplier
  $n^*Q\ (n^*)$ = 556 units to be produced every other order
  CTC_c = 845,000 CFA F per year for the worksite
  CTC_f = 1,257,100 CFA F per year for the supplier
  CTC = 2,102,100 CFA F per year for the both partners (worksite & supplier)

It is noted that when the management parameters are identified in coordination, the result is a lower overall cost. It is assumed that the partners (worksite - worksite or worksite - supplier) can agree on a fair mechanism for-profits sharing.

In conclusion, we provide a summary (Table 2) of actions to reduce the total cost of spare parts inventory management. This total management cost can be expressed as the sum of ordering, acquiring, storage, shortage, and maintenance costs. This table defines the various costs inherent to spare parts inventory management and recommendations for their reduction.

Code Description	**Cost Reduction Actions**
Order cost: Includes the costs (preparing, launching, monitoring reception of ordered items	* Orders streamlining and pooling * Using e-commerce tools to launch and monitor orders * Developing a partnership with suppliers
Cost of acquisition: Items purchase cost	* Orders pooling to benefit from economies of scale * Monitoring and benefiting from temporary discounts Carry out a regular search for new suppliers to extend one's supply pool and reduce one's supply purchase costs * Resort to remanufactured parts for items (sometimes)
Storage cost: varying cost, including all expenses resulting from an inventory item (rent insurance, taxes, interests, salaries). The varying storage cost of a unit over a given time horizon is about 20% to 60% of its cost of acquisition	* Reduce the quantities to be storage (pooling or inventory virtual centralization; inventory reduction thanks to similar components interchangeability) * Reduce the storage periods Gust- in-time, determining the order's placing optimal instant) Resort to remanufactured parts for items Provide
Shortage cost: Set of costs incurred after a Not full-filled request because the storage is empty. This cost may be difficult to access, but capacity loss, client compensation, and delivery postpone costs generally.	* Provide an alternative emergency supply (lateral transfer or loans from other worksites) * Reduce lead time (submission and order online) * Resort to functionally similar components (parts)
Cost of maintenance: it comprises the costs of all actions carried out to maintain or rehabilitate the equipment to good operation	* Improve the staff training * Prepare and organize maintenance actions (Computer-Assisted Maintenance Management "CAMM") * The planning system maintenance a link to actions with spare parts inventory management system * Ensure access to documentation (manufacturer or supplier web portal and appropriate tools)

CONCLUSION

Beyond its concern for traditional management policies' orthodoxy respect (presented from the beginning of our approach), this publication exposes the unrelenting question of collegial practice in spare parts inventory management in the construction sector. Like the easy access to technical documentation and the reduction in orders delivery time, other factors influencing inventory management, we could establish that *the isolated approach to inventory management in the construction equipment management sector is not recommended, as it is counterproductive*. Resort to inventory pooling and orders pooling is required under well-established conditions. The major opposition from this article is thus to recall the main recommendations, permitting construction equipment managers to succeed in making appreciable profits through a better collaborative and collegial approach in spare parts inventory management.

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