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The Role of Geometry in Stability of Tower-Like Mausoleums

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Abstract — Tower-Like mausoleums are single domed structures in the form of circle, truncated cone, cube, polygon or pyramid. The surface of such forms may appear cannellated. Also, the covering dome may be semi-circle or low-rise in shape, sometimes with an assertive appearance. Tower-like mausoleums are considered ancient structures in Persian-Islamic architecture and they have used to be common especially in Razi Style. In Razi Style mausoleums were built in the shape of tower, thole or tester, the best examples of which could be found in eastern Iran. Tower-Like mausoleums are remarkable structures symbolically. They are, in fact, brick-built structures with special architectural values. Form and shape of those play a determining role in their stability. In this respect, the studies so far carried out focused mainly on mausoleums in terms of decorations and architectural features. The present study, however, aimed to statically analyze the impact of different forms of mausoleums on their stability. To do so, relevant library and field studies were done. First, four tower-like mausoleums were introduced and examined. Also, these structures were explained architecturally and formally. The results indicated that elements including tower plan, floor dimensions, tower and presence or absence of jambs each affect the stability of mausoleums independently.

Keyword — Form and Geometry, Stability, Stability against Seismic, Tower-Mausoleums.

1. Introduction

In each period of architectural history there have been certain structures which have been symbolically outstanding; they have been vehicle for meanings or concepts. In addition to this aspect, however, there are other important noteworthy dimensions including height or elevation. The symbolic aspects as well as great height of minarets, domes and tower-mausoleums have made them outstanding landmarks. But, when the height factor is involved in construction, the stability and static considerations should be inevitably taken into account. Tower-mausoleums appear in a wide variety of forms each of which imposes a statistically different force. The present study aimed to examine the impact of form of a tower- mausoleum on its stability. In this regard, different tower-mausoleums in the north-eastern region of Iran were inspected and finally four quite formally different ones were selected. Characteristics of these four tower-mausoleums were addressed separately. The questions intended to be answered were as follows:

-Does the formal structure of a tower-mausoleum have any bearing on its stability?

-Which of the geometrical features of tower-mausoleums influence their stability?

-On what part of a tower-mausoleum does an earthquake impose the maximum force?

Through doing library and field studies, the formal plans of the selected tower mausoleums were prepared and drawn (by Auto Cad Software) with the help of analytical software (ANSYS (V:14.0)) and then examined statically; that is, the behavior of formal structure versus imposed tensions was observed.

2. Razi Style

Tower-mausoleums were especially common in Razi Style. It began in Ray. The salient features of the previous periods- delicate representations of Persian Style, magnificence of Parthian Style and detailed considerations of Khorasani Style-were all eclectically represented in this Style. Of the architectural structures in Razi period, tower-mausoleums and minarets are noteworthy. The construction of mausoleums in the period is, in fact, the continuation of Al-e- Bouye and Al-e-Ziar architectural styles [1]. The architectural and structural techniques used in these two styles are mainly under the influence of mausoleums built in the northern Iran. One of the key characteristics of Razi Style is construction of single-structure mausoleums [2].

3. Tower-Mausoleums

It seems that the idea of constructing mausoleums and monuments has been under the influence of Islamic customs and faiths [2]. Mausoleums often used to be built in the shape of square, pentagon, hexagon, octagon or circle with lobed or smooth external surface. The mausoleums were originally built as a landmark to guide the passers-by. Radkan Tower, Rasket Tower and Lajim Tower in northern Iran and Mil-e-Negar-e-Khorasan are the prime examples.

The founders of these towers often willed to be buried in the same (tower) mausoleum they had built [2].

The tower-mausoleums can be grouped in three distinct classes depending on their structural shape: circular, square and polygonal. Circular mausoleums are themselves of two kinds; those with simple circular plan and with lobed circular plan.

A. (Tower) mausoleums with simple circular plans: The whole structure usually consists of a cylindrical body and a conical dome and the internal plan is circular as well, for example, Chehel Dokhtaran mausoleum in Damghan.
B. (Tower) mausoleums with lobed external surface: the interior and exterior are circular in shape, for example, Toghrol Tower in Shahr-e-Rey (Fig. 1).
Square (tower) mausoleums: The body and plan of these structures are square in shape. The torque and the dome are hexagonal and pyramidal in shape respectively. Maraghe Red Dome is an example.
Polygonal (tower) mausoleums: Constructed on a hexagonal or a ten-sided polygon, the body is prismatic or ten-sided in shape. The dome also follows the body in shape [2]. The example of this is Gonbad-e-Ali (Ali dome) in Abarghoo (Fig. 2).

Fig.1. Toghrol tower in Shahr-e-Rey

Fig.2. Gonbad-e-Ali (Ali dome) in Abarghoo

4. Tower-Mausoleums Examined in this Study

4.1. Akhengan Tower

The monument is placed at 22 kilometers from the north of Mashad. The tower is a circular structure. The interior is hexagonal in cross section. There are eight half-columns around the tower [3]. The dome is conical with lobed surface. The dome drum and helmet are 13.7 m and 4.2 high respectively. The total height is 17.9m (Fig.3). According to Godard this monument belongs to Timurid era [4].

Fig.3. Plans and a picture of Akhengan Tower

4.2. Radkan-e-Gharbi (Western Radkan) Tower

According to the inscription written in Kufi style, mounted around the cylindrical body and under the conical dome of the structure, the construction of the monument started in 1016 A.D. and ended in 1020A.D.[5]. Peernia in his book Rah-o-Robat (Roads and Inns) observes that Rād and Rad in Pahlavi (the Persian language) means order and discipline; therefore, Rādkan (kān is a suffix indicating place) is a landmark showing the passers-by the correct way and direction [6]. Even today, the only landmark in the northeastern forests of Iran is the Kordkouy radkan. The tower is placed sensibly on a hill inviting the passers-by from far away [7]. The tower is circular in shape; externally (from the base) 9.9 m in diameter and 25 m in height (Fig. 4).

Fig.4. Plans and a picture of Radkan-e-Gharbi Tower

4.3. Gonbad-e-Ghabous (Ghabous Dome) Tower

Built on a man-created hilltop, Gonbad-e-Ghabous is placed in a village with the same name [8]. It is as high as 55.5m. Its circular plan has been divided by ten equally spaced triangular blades (Fig.5).
The blades are stretched vertically from the base towards the conical dome. But, the disappear near the projected lobe of the sharply-tilted dome [9].

Fig.5. Plans and a picture of Gonbad-e-Ghabous Tower

4.4. Lajim Tower

The tower is placed in Savad Kouh, Mazandaran Province. The only legible date written in Kufi on an inscription is 1022 A.D.[5].
It is the burial place of one of the distinguished persons of Al-e-Bavand (Bavand dynasty)[4]. The tower is cylinderical in shape. The dome, however, has lost the original appearance because of repeated restoration operations. Notwithstanding, the proportions between dome span, height and width of the walls are well-calculated (Fig.6). The façade of structure lacks any brick-made decoration. However, at the top of the tower cylinder there are several blind doors and crenels.

Fig.6. Plans and a picture of Lajim Tower

5. Stability Examination of the Tower-Mausoleum

After providing three-dimensional plan of the studied tower-mausoleums, they were analyzed by software ANSYS (v: 14.1). In modeling the towers, the element Solid 65 was applied (a four-knot pyramidal shape) so that the resulted cracks could be observed. In addition, to simplify the calculations, materials were considered homogeneous and their behavior was also supposed as linear [10]. The use of linear models as a previous step to nonlinear analysis, allows us in most of the cases to comprehend the real structural behaviour, with less computational cost and complexity, and they act as a guide for the subsequent nonlinear analyses [11]. The mechanical specifications of materials used in tower-mausoleums have been given in table1 [12]. In order to model a clamed support of the structure, the transitional freedom degrees of available knots were included in the foundation level [13]. Then, the linear static analysis was done under the weight load. The amounts of displacement were examined and the real tensions were compared with allowable ones. The real tensions included s_1, s_2, and s_3 where,

S_1= tensile stress, (max, 50,000)
S_2= shearing stress (max, 70,000)
S_3= compressive stress (max, 500,000)

To do the seismic analysis, Naghan earthquake (that was 6.5 Richter magnitude scale in1978A.D.) was considered as a reference point. Hence, the displacements and minimum/ maximum tensions were compared in different times.

Table (1) Mechanical specifications of materials

Density (ρ)	Poisson's ratio (υ)	Elasticity Modulus	Tensile Resistance	Compressive Resistance
1460 kg/m^3	0.17	0.5*109 N/m^2	175 N/m^2	3500 N/m^2

5.1. Akhengan Tower

After modeling the tower under the load of its own weight, the maximum displacement of 22.18mm at the tower apex was observed.

5.1.1. Examining tensions S_1, S_2 and S_3 of Akhengan Tower in comparison with allowable tension and suggesting the best possible preventive approach in the most critical points

According to table 2, the maximum tensile tension (which is present where the dome is connected to its cylindrical column shown by the red area (fig.7 (a)) is more than the permitted amount. Consequently, the connecting point may crack perpendicular to the tensile stress and therefore, tensile supportive elements should be applied. Conversely, the maximum shear stress, based on table 2, is less than that of the allowable stress and, as a result, there would be no shear crack in the area (Fig.7 (b)). Likewise, the maximum compressive stress in blue area (happening at the connecting point of tower body and floor) is less than that of the allowable stress and therefore there would be no cracks resulted from crushing and crippling (Fig.7(c)).

Table (2) Maximum and Minimum Values of Tensions in Akhengan Tower

Main Tensions	Maximum Value	Minimum Value
S_1	68472	-38914
S_2	50661	26105
S_3	289194	1501

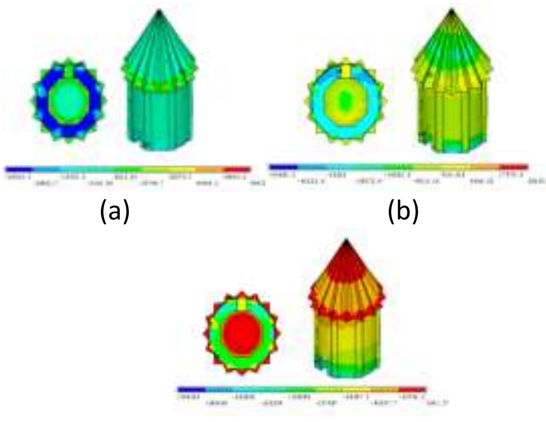


Fig.7. (a) Diagram of tensile tension, (b) diagram of shear tension and (c) diagram of compressive tension in Akhengan Tower.

5.1.2. Results of computerized seismic analysis

The seismic analysis was carried out with reference to Naghan earthquake using 252 time-acceleration value pairs (table3). In doing so, tensions and displacements were examined in different times. The graphic results at the most critical time duration, which was two seconds for Naghan earthquake, were examined.

The maximum displacement resulted from earthquake can be observed at the end of second two at the dome apex (0.014232). As table 3 shows, the maximum tensile stress S_1 is 11 times greater than that of the allowable value. Similarly, the minimum tensile stress is 2.5 times greater than that of the allowable value. In other words, the tensile stress of the whole structure is greater than that of the allowable value. In addition, the maximum shear stress S_2 is 1.5 times more than that of the permitted value and its minimum value is 2.5 times more than the permitted shear stress value. In a similar token, the maximum compressive value S_3 is 2 times greater than that of the permitted compressive stress value. The minimum compressive stress, however, is less than that of the permitted value.

Table (3) Results of tensions and displacements in seismic analysis of Akhengan Tower

Step	Time (s)	Dmax (m)	S_1 min (N/m^2)	S_1 max (N/m^2)	S_2 min (N/m^2)	S_2 max (N/m^2)	S_3 min (N/m^2)	S_3 max (N/m^2)
1	0.02	0.002218	-38914	-68472	-50661	26105	-289194	1501
60	1.2	0.00396	-55592	91103	-72946	29580	-421653	1976
100	2	0.014232	-113646	588553	-179528	96604	-.103E+07	77856
150	3	0.013958	-120797	541923	-159174	96352	-.106E+07	68563
252	5.04	0.006123	-72449	109129	-96726	36086	-556693	14571

5.2. Radkan-e-Gharbi (Western Radkan) Tower

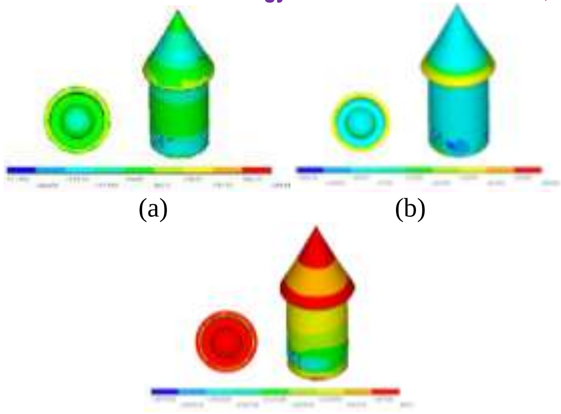
After modeling the tower under the load of its own weight, the maximum displacement of 50.32mm at the tower apex was observed.

5.2.1. Examining tensions S_1, S_2 and S_3 of Radkan-e-Gharbi Tower in comparison with allowable tension and suggesting the best possible preventive approach in the most critical points

According to table 4, the maximum tensile tension (which is present where the dome is connected to its cylindrical column shown by the red area in fig.8 (a)) is more than the permitted amount. Consequently, the connecting point may crack perpendicular to the tensile stress and therefore, tensile supportive elements should be applied. Conversely, the maximum shear stress, based on table 4, is less than that of the allowable stress and, as a result, there would be no 45 degree shear crack in the area (Fig.8 (b)). Likewise, the maximum compressive stress in blue area (happening at the connecting point of tower body and floor) is less than that of the allowable stress and therefore there would be no cracks resulted from crushing and crippling (Fig.8 (c)).

Table (4) Maximum and Minimum Values of Tensions in Radkan-e-Gharbi Tower

Main Tensions	Minimum Value	Maximum Value
S_1	60642	-26214
S_2	51750	49699
S_3	387508	4801



(a) (b) (c)

Fig.8. (a) Diagram of tensile tension, (b) diagram of shear tension and (c) diagram of compressive tension in Radkan-e-Gharbi Tower

5.2.2. Results of computerized seismic analysis

The seismic analysis was carried out with reference to Naghan earthquake using 252 time-acceleration value pairs (table5).

Table (5) Results of tensions and displacements in seismic analysis of Radkan-e-Gharbi Tower

step	Time(s)	Dmax (m)	S_1 min (N/m^2)	S_1 max (N/m^2)	S_2 min (N/m^2)	S_2 max (N/m^2)	S_3 min (N/m^2)	S_3 max (N/m^2)
1	0.02	0.005032	-24214	60642	-51750	49699	-387508	4801
40	0.8	0.010017	-25187	74247	-53005	50775	-418417	5045
100	2	0.046198	-39915	480969	-116862	101779	-919480	4648
200	4	0.005096	-26102	61379	-52053	49491	-385175	4813
252	5.04	0.019238	-29667	109219	66403	64461	-504879	4252

The maximum displacement resulted from earthquake can be observed at the end of second two at the dome apex (0.046198). As table 5 shows, the maximum tensile stress S_1 is 10 times greater than that of the allowable value. Similarly, the minimum tensile stress is 0.8 times greater than that of the allowable value. In other words, the tensile stress of the whole structure is greater than that of the allowable value. In addition, the maximum shear stress S_2 is 1.6 times more than that of the permitted value and its minimum value is 1.5 times more than the permitted shear stress value. In a similar token, the maximum compressive value S_3 is 1.8 times greater than that of the permitted compressive stress value. The minimum compressive stress, however, is less than that of the permitted value.

5.3. Gonbad-e-Ghabous Tower

After modeling the tower under the load of its own weight, the maximum displacement of 34.6mm at the tower apex was observed.

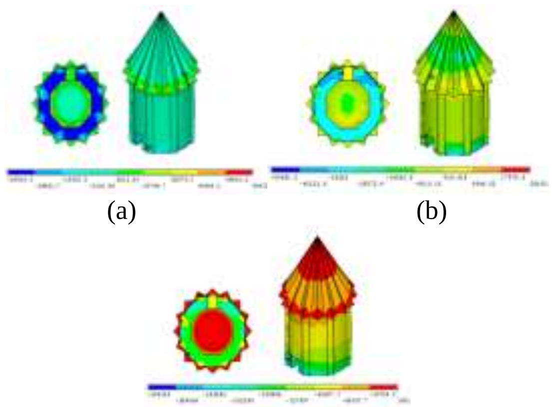
5.3.1. Examining tensions S_1, S_2 and S_3 of Gonbad-e-Ghabous Tower in comparison with allowable tension and suggesting the best possible preventive approach in the most critical points

According to table 6, the maximum tensile tension (which is present where the dome is connected to its cylindrical column shown by the red area in fig.9 (a)) is more than the permitted amount. Consequently, the connecting point may crack perpendicular to the tensile stress and therefore, tensile supportive elements should be applied. Conversely, the maximum shear stress, based on table 6, is more than that of the allowable stress and, as a result, there would be 45 degree shear crack in the area (Fig.9 (b)).So, for resolving this problem, the possible choices are; injecting concrete to the cracks or using of same metal belts. The maximum compressive stress in blue area (happening at the connecting point of tower body and floor) is more than that of the allowable stress and therefore there would be no cracks resulted from crushing and crippling (Fig.9 (c)).

For resolving this issue, concrete or metal belts would be used for preventing of strain in X axis direction.

Table (6) Maximum and Minimum Values of Tensions in Gonbad-e-Ghabous Tower

Minimum Value	Maximum Value	Main Tensions
S_1	60149	-132614
S_2	145352	18413
S_3	757830	1204



(a) (b) (c)

Fig.9. (a) Diagram of tensile tension, (b) diagram of shear tension and (c) diagram of compressive tension in Gonbad-e-Ghabous Tower.

5.3.2. Results of computerized seismic analysis

The seismic analysis was carried out with reference to Naghan earthquake using 252 time-acceleration value pairs (table7) [14].

The maximum displacement resulted from earthquake can be observed at the end of second two at the dome apex (0.242921). As table 7 shows, the maximum tensile stress S_1 is 20 times greater than that of the allowable value. Similarly, the minimum tensile stress is 2.5 times greater than that of the allowable value. In other words, the tensile stress of the whole structure is greater than that of the allowable value. In addition, the maximum shear stress S_2 is 2 times more than that of the permitted value and its minimum value is 2 times more than the permitted shear stress value. In a similar token, the maximum compressive value S_3 is 2 times greater than that of the permitted compressive stress value. The minimum compressive stress, however, is less than that of the permitted value.

Table (7) Results of tensions and displacements in seismic analysis of Gonbad-e-Ghabous Tower

step	Time(s)	Dmax (m)	$S_{1\,min}$ (N/m^2)	$S_{1\,max}$ (N/m^2)	$S_{2\,min}$ (N/m^2)	$S_{2\,max}$ (N/m^2)	$S_{3\,min}$ (N/m^2)	$S_{3\,max}$ (N/m^2)
1	0.02	0.149E-04	-5.415	1341	-148.274	147.967	-1335	2.932
49	0.98	0.02794	-27268	218559	-34788	34845	-217450	27403
100	2	0.242921	-125081	0.103E+07	-156490	152606	-0.104E+07	125220
200	4	0.034669	-48980	0.100E+07	-182296	188342	0.104E+07	43849
251	5.02	0.116765	-243512	0.250E+07	-374091	369364	0.254E+07	237792

5.4. Lajim Tower

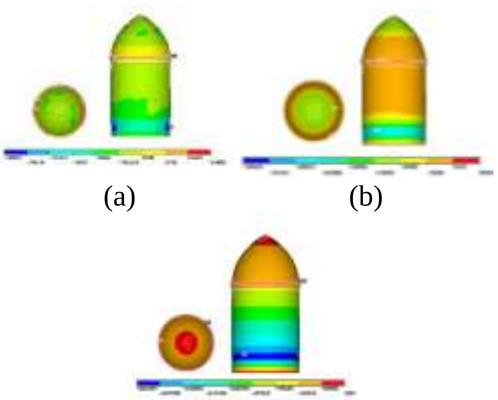
After modeling the tower under the load of its own weight, the maximum displacement of 3.47mm at the tower apex was observed.

5.4.1. Examining tensions S_1, S_2 and S_3 of Lajim Tower in comparison with allowable tension and suggesting the best possible preventive approach in the most critical points

According to table 8, the maximum tensile tension (which is present where the dome is connected to its cylindrical column shown by the red area in fig.10 (a)) is more than the permitted amount. Consequently, the connecting point may crack perpendicular to the tensile stress and therefore, tensile supportive elements should be applied. Conversely, the maximum shear stress, based on table 8, is less than that of the allowable stress and, as a result, there would be no shear crack in the area (Fig.10 (b)). Likewise, the maximum compressive stress in blue area (happening at the middle of tower body) is less than that of the allowable stress and therefore there would be no cracks resulted from crushing and crippling (Fig.10 (c)).

Table (10) Maximum and Minimum Values of Tensions in Lajim Tower

Minimum Value	Maximum Value	Main Tensions
S_1	15036	-18974
S_2	39467	6678
S_3	1237	221857



(a) (b) (c)

Fig.10. (a) Diagram of tensile tension, (b) diagram of shear tension and (c) diagram of compressive tension in Lajim Tower.

5.4.2. Results of computerized seismic analysis

The seismic analysis was carried out with reference to Naghan earthquake using 252 time-acceleration value pairs (table9). The maximum displacement resulted from earthquake can be observed at the end of second two at the dome apex (0.026767). As table 9 shows, the maximum tensile stress S_1 is 11.5 times greater than that of the allowable value. Similarly, the minimum tensile stress is 3.5 times greater than that of the allowable value. In other words, the tensile stress of the whole structure is greater than that of the allowable value. In addition, the maximum shear stress S_2 is 3 times more than that of the permitted value and its minimum value is 1.5 times more than the permitted shear stress value. In a similar token, the maximum compressive value S_3 is 2 times greater than that of the permitted compressive stress value. The minimum compressive stress, however, is less than that of the permitted value.

Table (9) Results of tensions and displacements in seismic analysis of Lajim Tower

step	Time (s)	Dmax (m)	$S_{1\ min}$ (N/m^2)	$S_{1\ max}$ (N/m^2)	$S_{2\ min}$ (N/m^2)	$S_{2\ max}$ (N/m^2)	$S_{3\ min}$ (N/m^2)	$S_{3\ max}$ (N/m^2)
1	0.02	0.0347	-50890	16051	-51716	11177	-253326	1624
60	1.2	0.007091	-80897	34840	-87659	9569	-434746	2337
100	2	0.026767	-173359	577296	-208189	106592	-.105E+07	56437
195	3.9	0.015478	-120568	228728	-139505	39257	-700473	18765
252	5.04	0.011312	-101539	102766	-114353	20971	-571128	10475

6. Conclusion

Given the results obtained, the formal structure and shape of the tower plan directly affect the tower stability. Among the factors studied presence or absence of symmetry, ribbed forms as well as dome formal structure affect the stability of tower mausoleums. In sum, the results analyses indicated that tower mausoleums were quite resistant against the weight load. Also, the simpler the geometrical forms of the mausoleums, the more stable they would be. Radkan-e-Gharbi Tower (section2-4) and Lajim (section4-4) were simpler in terms of geometrical forms compared with Akhengan Tower (section1-4) and Gonad-e-Ghabous (section3-4). The presence of ten blades in Gonbad-e-Ghabous and of half-columns in Akhengan Tower, both symmetrically constructed in the towers, has made them more stable. Of course, due to fragility of materials used, mainly bricks, a part of tensile cracks may be resulted from bending of materials.

In addition to form and geometrical structure, symmetry also plays a key role in tower stability. Symmetry distributes mass and forces homogeneously. Based on the results from seismic analyses, most tension cases took place at the connecting point of dome and main body. Also, cracks may happen at the base of walls. Therefore, in order to prevent such tensions, walls should be strengthened (by using elements compatible with historical monuments) against tensile tensions in perpendicular to points where cracks may happen.

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