

UNSTEADY FREE CONVECTION FLOW WITH FLOW CONTROL PARAMETER

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Abstract — The unsteady incompressible free convection boundary layer equations in two dimensions, with heat transfer have been solved numerically using finite difference method. The unsteady free convection flow with flow control parameter effect is presented in this work. The governing second order differential equations are derived from the fluid motion (Navier-Stokes equation) and heat conduction (energy equation) equations. Mathematical formulation of boundary layer equations have been non-dimensionalized by using dimensionless variables. These non-dimensional boundary layer equations are non-linear and partial differential equations and are solved by finite difference method. The fluid particles such as air ($P_r = 0.71$), water at 20°C ($P_r = 0.70$) and salt water ($P_r = 1.0$) are considered. In different Eckert numbers and for various Grashoff numbers as well as Prandtl numbers, the results of velocity and temperature are displayed in the form of level curves for both the temperature and velocity function.

Keyword — Eckert number, Prandtl number, Grashoff number, Thermal expansion coefficient.

1. INTRODUCTION

In the studies related to heat transfer, considerable effort has been directed towards the convective model, in which the relative motion of the fluid provides an additional mechanism for the transfer of energy and material, the later being a more important consideration in cases where mass transfer, due to concentration difference occurs. Convection is inevitably coupled with the conduction mechanism, the eventual transfer of energy from one fluid element to another in its neighborhood is thorough conduction. The usual way is to first consider heat transfer without mass transfer, and present at a large stage a briefing of similarities and differences between heat transfer and mass transfer, with some specific examples of mass transfer applications.

There are complex problems where heat and mass transfer processes are combined with chemical reactions, as in combustion. In processes such as drying, evaporation at the surface water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. In many of these

processes, the interest lies in the determination of the total energy transfer, although in processes such as drying, the interest lies mainly in the overall mass transfer for moisture removal. Natural convection processes involving the combined mechanism are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, in many industrial applications involving solutions and mixtures in the absence of externally induced flow and in many chemical processing systems.

The natural convection boundary-layer flow generated in the fluid adjacent to a heated, vertical semi-infinite plate is one of the fundamental flows in heat and mass transfer. Most studies have examined the fully developed flow with relatively few investigations of the transient response to impulsive heating. The transient response of a stably stratified fluid adjacent to a vertical semi-infinite plate subjected to an impulsively applied constant heat flux boundary condition will be investigated for flow Prandtl number $P_r < 1$. Flows with $P_r < 1$ have many important applications; in particular liquid metals with $P_r < 1$ have been used for rapid cooling in nuclear reactors (1972, 1991, 1999) [1-3].

Semi-analytic solutions for the steady flow adjacent to a constant heat flux vertical semi-infinite plate have been obtained by a number of workers by reducing the governing equations to a set of ordinary differential equations which are then integrated numerically (1956, 1969) [4,5]. Such an approach can be used to obtain the solution at specific prandtl number, but does not provide scaling, and has also been shown to have difficulty dealing with very small Prandtl number (1969) [5]. Additional investigations of the steady flow have been carried out using the integral method of Karman-Pohlhausen and using singular perturbation techniques (1974) [6]. These results did not provide explicit Prandtl number scaling. *Park and Carey* (1985)[7] combined a matched asymptotic expansion technique with a explicit finite-difference scheme to investigate the transient natural convection flow near a vertical surface at low Prandtl number; *Sammounda, Beighith and Surry* (1999)[3] used a finite element simulation to investigate the transient natural convection of low Prandtl number fluids in a heated cavity; the low Prandtl number convection in volumetrically heated rectangular enclosures with different aspect ratios was explored by direct numerical two-dimensional simulation by *Piazza, Ciofalo and Arcidiacono* (2000,2001) [8-9].

The investigation cited above focused on the unsteady natural convection boundary-layer flow in an initially quiescent homogeneous ambient fluid. However, in many problems of practical interest the ambient fluid is at a non-uniform temperature, and is typically stably stratified. *Park and Hyun* (1998, 2002)[10,11] investigated the transient adjustment process of an initially stationary and stably stratified fluid in a square container with highly conducting boundary walls, while *Chamkha* (2002)[12] investigated the laminar. However there has been no study that examines the Prandtl number effect on the flow behavior adjacent to an evenly heated semi-infinite plate with a stratified ambient for low Prandtl numbers for the complete problem of start-up, transition and full development, which motivates the present investigation. In this study, we will develop various scaling laws for the dominant parameters characterizing the transient behavior of an unsteady natural convection boundary-layer flow of an initially linearly-stratified Newtonian fluid with $Pr < 1$ adjacent to a semi-infinite vertical plate heated with a uniform flux, using the techniques detailed by *Patterson and Imberger* (1980) [13] and *Bejan* (1995)[14]. The remainder of this study is organized as follows. The equation of conservation of mass or equation of motion for inviscid fluid, Navier-Stokes equation in vector form with Cartesian coordinate, equations of energy and finally Mass transfer have been derived and discussed. Concept of boundary layer in two dimensional flows, derivation of thermal boundary layer with concentration, derivation of thermal boundary layer equation in terms of energy equation and the model has been formulated. Finally, unsteady free convection flow with flow control parameter has been discussed graphically.

2. MATHEMATICAL MODEL OF FLOW

Introducing the Cartesian co-ordinate system the x-axis is chosen along the plate in the direction of flow and the y-axis is normal to it. Initially we consider that the plate as well as the fluid is at the same temperature $T(T_\infty)$. Also it is assumed that the fluid and the plate is at rest after that the plate is to be moving with a constant velocity U_0 in its own plane and instantaneously at time $t > 0$, the temperature of the plate raised to $T_w (> T_\infty)$ which is there after maintained constant, where T_w is temperature at the wall and T_∞ is the temperature of the species far away from the plate. The physical model of the study is furnished in the flowing Figure 1

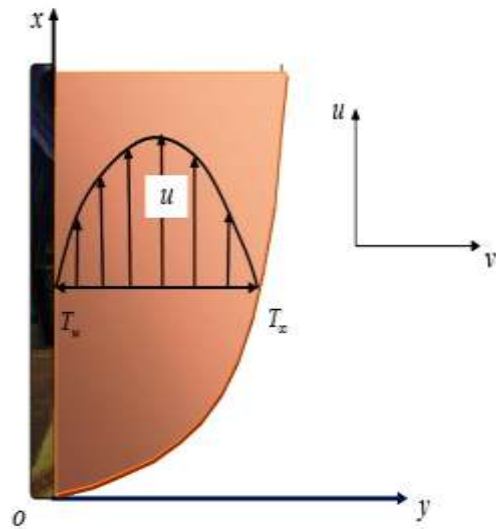


FIG. 1. Physical configuration and co-ordinate system of model.

Within the framework of the above stated assumptions with reference to the generalized equations described in chapter 4, the equation relevant to the transient two dimensional problem are governed by the following system of couple non-linear partial differential equations

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_\infty) \quad (2)$$

Energy equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

With corresponding initial and boundary conditions

$$t = 0, u = 0, v = 0, T \rightarrow T_\infty, \quad \text{everywhere}$$

$$(4) t \geq 0, \begin{cases} u = 0, v = 0, T \rightarrow T_\infty \text{ as } x = 0 \\ u = 0, v = 0, T = T_w \text{ at } y = 0 \\ u = 0, v = 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{cases}$$

(5)

Where,

x, y = Cartesian Co-ordinate system;

u, v = x, y component of flow velocity respectively is

the local acceleration due to gravity;

ν = the kinematic viscosity;

ρ = the density of the fluid;

k = the thermal conductivity;

C_p = the specific heat at constant pressure.

2.1 Mathematical Formulation

Since the solutions of the governing equations (1) to (3) under the initial conditions (4) and boundary condition

(5) will be based on a finite difference method. It is required to make the equations dimensionless. For this purpose we introducing following dimensionless variable. Let

$$X = \frac{xU_0}{\nu}; \quad Y = \frac{yU_0}{\nu}; \quad U = \frac{u}{U_0};$$

$$V = \frac{v}{U_0}; \quad \tau = \frac{tU_0^2}{\nu}; \quad \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}$$

$$, \quad x = \frac{X\nu}{U_0}, \quad y = \frac{Y\nu}{U_0}$$

Using these relations, we have the following derivatives,

$$\frac{\partial u}{\partial t} = \frac{U_0 \partial U}{\frac{\nu}{U_0} \partial \tau} = \frac{U_0^3 \partial U}{\nu \partial \tau},$$

$$\frac{\partial u}{\partial x} = \frac{U_0 \partial U}{\frac{\nu}{U_0} \partial X} = \frac{U_0^2 \partial U}{\nu \partial X},$$

$$\frac{\partial u}{\partial y} = \frac{U_0 \partial U}{\frac{\nu}{U_0} \partial Y} = \frac{U_0^2 \partial U}{\nu \partial Y},$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{U_0^2 \partial U}{\nu \partial Y} \right) = \frac{U_0^2}{\nu} \frac{\partial^2 U}{\partial Y \frac{\nu}{U_0} \partial Y} = \frac{U_0^3 \partial^2 U}{\nu^2 \partial Y^2}$$

$$\frac{\partial v}{\partial y} = \frac{U_0 \partial V}{\frac{\nu}{U_0} \partial Y} = \frac{U_0^2 \partial V}{\nu \partial Y},$$

$$\frac{\partial T}{\partial t} = \frac{(T_w - T_\infty) \partial \bar{T}}{\frac{\nu}{U_0^2} \partial \tau} = \frac{U_0^2 (T_w - T_\infty) \partial \bar{T}}{\nu \partial \tau},$$

$$\frac{\partial T}{\partial x} = \frac{(T_w - T_\infty) \partial \bar{T}}{\frac{\nu}{U_0} \partial X} = \frac{U_0 (T_w - T_\infty) \partial \bar{T}}{\nu \partial X},$$

$$\frac{\partial T}{\partial y} = \frac{(T_w - T_\infty) \partial \bar{T}}{\frac{\nu}{U_0} \partial Y} = \frac{U_0 (T_w - T_\infty) \partial \bar{T}}{\nu \partial Y},$$

Now putting these values in equations (1),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{U_0^2 \partial U}{\nu \partial X} + \frac{U_0^2 \partial V}{\nu \partial Y} = 0$$

$$\Rightarrow \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad ; \text{ satisfying the continuity equation.}$$

Equation (2) becomes,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_\infty)$$

$$\Rightarrow \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \frac{g\beta\nu}{U_0^3} (T_w - T_\infty) \bar{T}$$

$$\Rightarrow \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + G_r \bar{T}$$

Here, Grashoff Number,

$$G_r = \frac{g\beta\nu}{U_0^3} (T_w - T_\infty)$$

$$= \frac{[LT^{-2}][L^2T^{-1}][\theta^{-1}][\theta]}{L^3T^{-3}} = 1$$

And thermal expansion coefficient,

$$\beta = \frac{\text{length}}{\text{length} \times \text{temperature difference}}$$

$$= [\theta^{-1}]$$

Again, equation (3) becomes,

$$\Rightarrow \frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \bar{T}}{\partial Y^2} + E_c \left(\frac{\partial U}{\partial Y} \right)^2$$

Here,

$$P_r = \text{Prandtl number} = \frac{\rho\nu C_p}{k}$$

$$E_c = \text{Eckerd number} = \frac{U_0^2}{C_p (T_w - T_\infty)}$$

Again, we prove that P_r and E_c are dimensionless parameter.

$$P_r = \frac{\rho\nu C_p}{k}$$

$$= \frac{[L^2T^{-1}][ML^{-3}][L^2T^{-2}\theta^{-1}]}{[MLT^{-3}\theta^{-1}]} = 1$$

$$E_c = \frac{U_0^2}{C_p (T_w - T_\infty)} = \frac{[L^2T^{-2}]}{[L^2T^{-2}\theta^{-1}][\theta]}$$

Now we get the following non-linear coupled partial differential equation in terms of dimensionless variables,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = G_r \bar{T} + \frac{\partial^2 U}{\partial Y^2}$$

$$\frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \bar{T}}{\partial Y^2} + E_c \left(\frac{\partial U}{\partial Y} \right)^2$$

Also the associate initial and boundary conditions becomes,

$$\begin{aligned} \tau = 0, \quad U = V = 0, \quad \bar{T} = 0 \quad \text{Everywhere,} \\ \tau \geq 0, \quad U = V = 0, \quad \bar{T} = 1 \quad \text{at } Y = 0 \\ U = V = 0, \quad \bar{T} = 0 \quad \text{at } X = 0 \\ U = V = 0, \quad \bar{T} = 0 \quad \text{as } Y \rightarrow \infty \end{aligned}$$

3. NUMERICAL SOLUTIONS

From the concept of the above discussion for simplicity the explicit finite difference method has been used to solve equations subject to the conditions which given. To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes. Where X -axis is taken along the plate and Y -axis is normal to the plate. Here we consider that the plate height $X_{\max} = 100$.

i.e., X varies from 0 to 100 and regard $Y_{\max} (= 25)$ as corresponding to $Y \rightarrow \infty$.

i.e., Y varies from 0 to 25. There are $m = 125$ and $n = 125$ grid spacing in the X and Y direction respectively as shown in figure 2.

It is assumed that $\Delta X, \Delta Y$ are constant mesh sizes along X and Y direction respectively and taken as follows

$$\Delta X = 2.0 \quad (0 \leq x \leq 100)$$

$$\Delta Y = 0.5 \quad (0 \leq y \leq 25)$$

With the smaller time step, $\Delta \tau = 0.05$

We can draw the chart for finite difference method.

Here, $U', V', T' & C'$ denote the values of $U, V, \bar{T} & \bar{C}$ at the end of a time step respectively, using the finite difference approximation. We have,

$$\left(\frac{\partial U}{\partial \tau} \right)_{i,j} = \frac{U'_{i,j} - U_{i,j}}{\Delta \tau}; \quad \left(\frac{\partial U}{\partial X} \right)_{i,j} = \frac{U_{i,j} - U_{i-1,j}}{\Delta X};$$

$$\left(\frac{\partial U}{\partial Y} \right)_{i,j} = \frac{U_{i,j+1} - U_{i,j}}{\Delta Y}; \quad \left(\frac{\partial V}{\partial Y} \right)_{i,j} = \frac{V_{i,j} - V_{i,j-1}}{\Delta Y};$$

$$\left(\frac{\partial \bar{T}}{\partial \tau} \right)_{i,j} = \frac{\bar{T}'_{i,j} - \bar{T}_{i,j}}{\Delta \tau}; \quad \left(\frac{\partial \bar{T}}{\partial X} \right)_{i,j} = \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X};$$

$$\left(\frac{\partial^2 \bar{T}}{\partial Y^2} \right)_{i,j} = \frac{\bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1}}{\Delta Y^2}.$$

(6)

From the system partial difference equations with substituting the above relations into the corresponding

differential equation we obtain an approximate set of finite difference equations.

$$\begin{aligned} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ \Rightarrow \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + \frac{V_{i,j} - V_{i-1,j}}{\Delta Y} &= 0 \end{aligned} \quad (7)$$

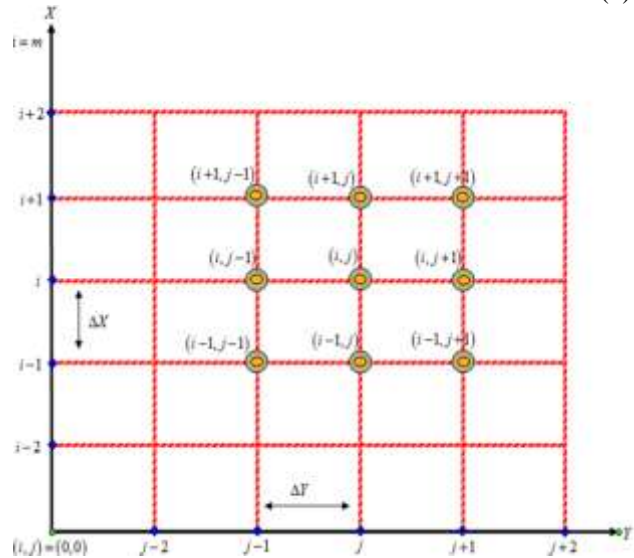


FIG. 2. The Finite difference space grid.

$$\begin{aligned} \frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= G_r \bar{T} + \frac{\partial^2 U}{\partial Y^2} \\ \Rightarrow \frac{U'_{i,j} - U_{i,j}}{\Delta \tau} + U \frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V \frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \\ &= G_r \bar{T} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta Y^2} \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial \bar{T}}{\partial \tau} + U \frac{\partial \bar{T}}{\partial X} + V \frac{\partial \bar{T}}{\partial Y} &= \frac{1}{P_r} \frac{\partial^2 \bar{T}}{\partial Y^2} + E_c \left(\frac{\partial U}{\partial Y} \right)^2 \\ \Rightarrow \frac{\bar{T}'_{i,j} - \bar{T}_{i,j}}{\Delta \tau} + U \frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} + V \frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta Y} \\ &= \frac{1}{P_r} \frac{\bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1}}{\Delta Y^2} + E_c \left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \right)^2 \end{aligned} \quad (9)$$

Here the changing initial and boundary conditions are

$$U^0_{i,j} = 0, \quad V^0_{i,j} = 0, \quad \bar{T}^0_{i,j} = 0 \quad (10)$$

$$U^n_{0,j} = 0, \quad V^n_{0,j} = 0, \quad \bar{T}^n_{0,j} = 0$$

$$U^n_{i,0} = 0, \quad V^n_{i,0} = 0, \quad \bar{T}^n_{i,0} = 1 \quad \text{Where } L \rightarrow \infty \quad (11)$$

$$U^n_{i,L} = 0, \quad V^n_{i,L} = 0, \quad \bar{T}^n_{i,L} = 0$$

Here the subscripts i and j designate the grid points with x and y coordinates respectively and superscript n represents a value of time, $\tau = n\Delta \tau$ where $n = 0, 1, 2, 3, \dots$. From the initial condition

(6), the values of U, \bar{T} are known at $\tau = 0$. During any one time step, the coefficients $U_{i,j}$ and $V_{i,j}$ appearing in (6) are created as constants. Then at the end of any time-step $\Delta\tau$ the temperature \bar{T}' , the new velocity U' , the new induced velocity field V' at all interior nodal points may be obtained by successive applications of equations (7), (8), (9) respectively. This process is repeated in time and provided the time-step is sufficiently small, U, V, \bar{T} should eventually converge to values which approximate the steady-state solution of equation (7)-(9). These converged solutions are shown graphically.

4. RESULTS AND DISCUSSION

Unsteady free convection flow with flow control parameter past a finite vertical plate in presence of heat transfer has been studied. The governing equations of the flow field are solved employing finite difference method and approximate solutions are obtained for velocity field and temperature field. The main goal of the computation is to obtain the steady-state solution for the non-dimensional velocity U and temperature \bar{T} for different values of Grashoff number G_r at Prandtl number $P_r = 0.71$ and Eckert number $Ec = 0.01$, different values of Prandtl number P_r at Grashoff number $G_r = 1.0$ and Eckert number $Ec = 0.01$ and different values of Eckert number Ec at Grashoff number $G_r = 1.0$ and Prandtl number $P_r = 0.71$. For this purpose computations have been carried out up to dimensionless time $\tau = 80$. The results of the computations show little changes for $\tau = 10$ to $\tau = 60$. But after $\tau = 60$ to $\tau = 80$ the results remain approximately same. Thus the solution for $\tau = 60$ are essentially steady-state solutions. Along with the steady-state solution for the transient values of U and T is shown in FIG (3-14) for time $\tau = 10$ and 60 respectively. The most important fluids are atmospheric air, water and saltwater. So the results are limited to $P_r = 0.71$ (Prandtl number for air and many other gases), $P_r = 7.00$ (Prandtl number for water at $20^\circ C$) and $P_r = 1.0$ (Prandtl number for saltwater).

4.1 Velocity Field

The velocity of flow field is found to change more or less with the variation of the flow parameters. The major factors affecting the velocity of the flow field are Grashoff number for heat transfer G_r , Eckert number Ec and Prandtl number P_r . The effects of these parameters on the velocity field have analyzed with the help of FIG (3-11).

In between FIG [3, 4] we depict the effect of Grashoff number for heat transfer on velocity. The values of

another parameter Eckert number ($Ec = 0.01$) and Prandtl number $P_r = 0.71$ are constant. The Grashoff number for heat transfer is found to enhance (upward) velocity at all points due to the action of free convection in the flow field. FIG [5, 6] presents the effect of Prandtl number on velocity of the flow field. The presence of heavier Prandtl number in the flow field is found to decelerate velocities at all points. In FIG [7, 8] we depict the effect of Eckert number on the velocity of the flow field. It is obvious that an increase in the Eckert number results in an increase in the velocity of the flow field at all points.

4.2 Temperature Field

The temperature of the flow field suffers a substantial change with the variation of the flow parameters such as Prandtl number P_r , Eckert number Ec and Grashoff number G_r . These variations are shown in FIG (9-14).

FIG [9, 10] represents the effect of Prandtl number against y on the temperature field keeping other parameters of the flow field constant. It is observed that an increase in the Prandtl number decreases the temperature of the flow field at all points. In FIG [11, 12], we analyze the effect of Eckert number on the temperature field. A growing Eckert number is found to increase the temperature of the flow field at all points. FIG [13, 14] depict remarkable effect of Grashoff number against y on the temperature field keeping other parameters of the flow field constant. It is obvious that an increase in the Grashoff number results in an increase in the temperature of the flow field.

The effective changes are follows.

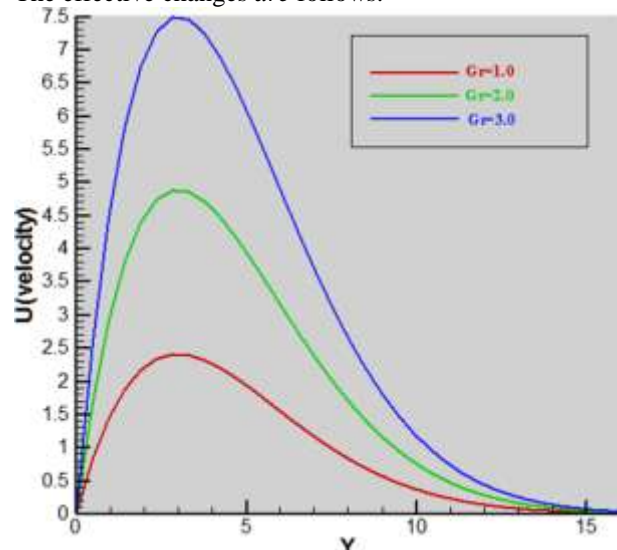


FIG. 3. Velocity profile for different values of Grashoff number G_r with fixed Eckert number $Ec = 0.01$ and Prandtl number $P_r = 0.71$ at a time $\tau = 10$.

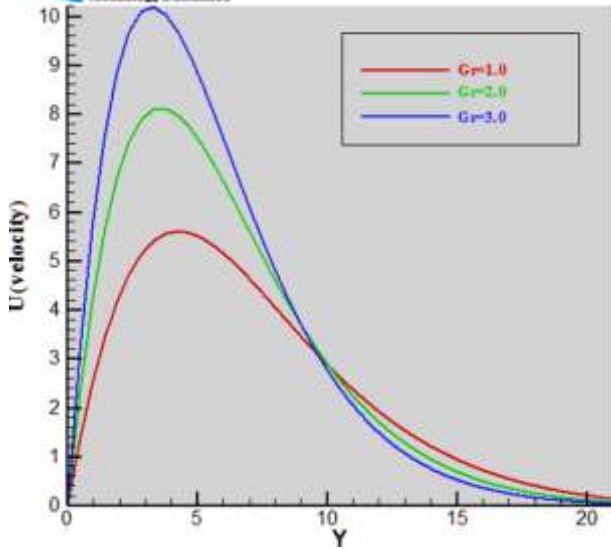


FIG. 4. Velocity profile for different values of Grashoff number G_r , with fixed Eckert number $E_c = 0.01$ and Prandtl number $P_r = 0.71$ at a time $\tau = 60$.

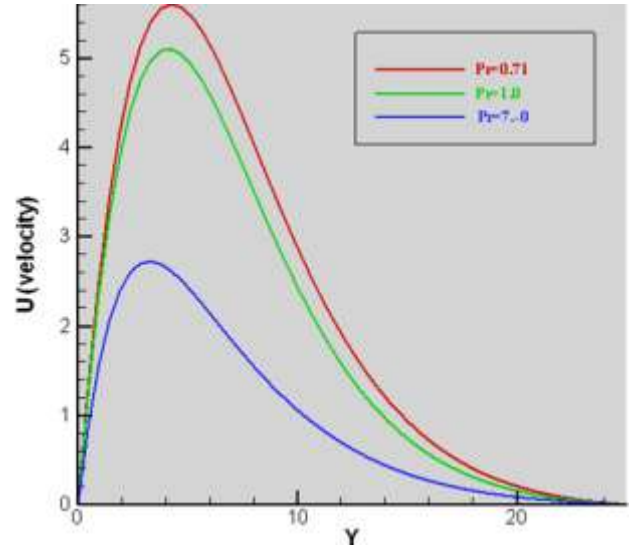


FIG. 6. Velocity profile for different values of Prandtl number P_r with fixed Eckert number $E_c = 0.01$ and Grashoff number $G_r = 1.0$ at a time $\tau = 60$.

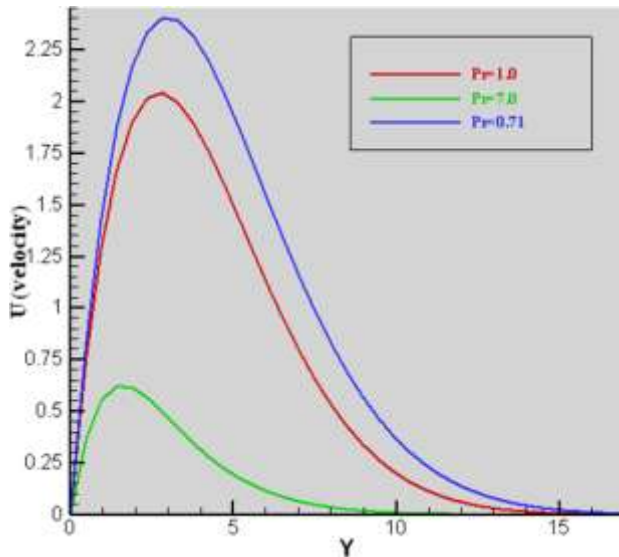


FIG. 5. Velocity profile for different values of Prandtl number P_r with fixed Eckert number $E_c = 0.01$ and Grashoff number $G_r = 1.0$ at a time $\tau = 10$.

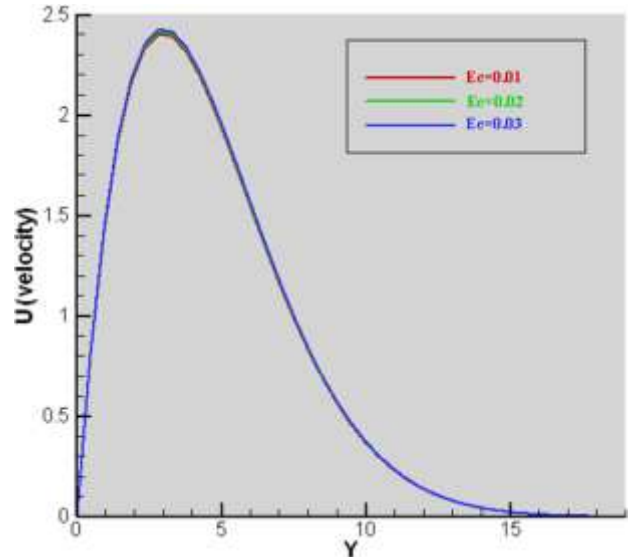


FIG. 7. Velocity profile for different values of Eckert number E_c with fixed Prandtl number $P_r = 0.71$ and Grashoff number $G_r = 1.0$ at a time $\tau = 10$.

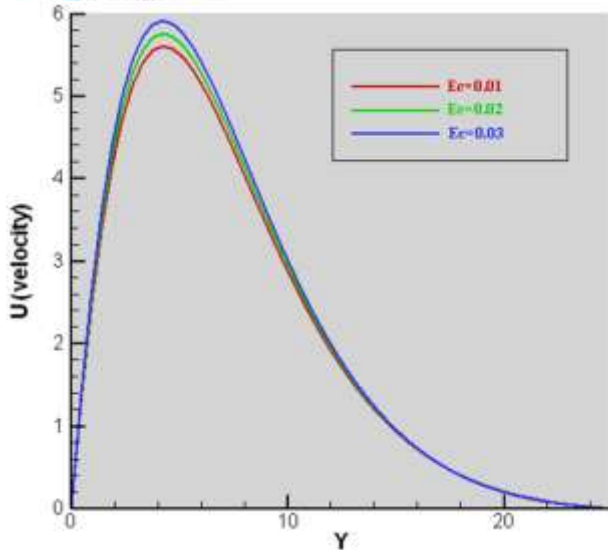


FIG. 8. Velocity profile for different values of Eckert number E_c with fixed Prandtl number $P_r = 0.71$ and Grashoff number $G_r = 1.0$ at a time $\tau = 60$.

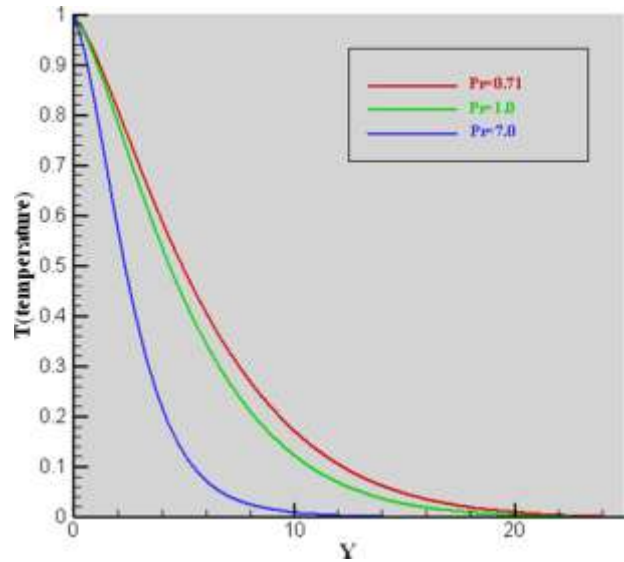


FIG. 10. Temperature profile for different values of Prandtl number P_r with fixed Eckert number $E_c = 0.01$ and Grashoff number $G_r = 1.0$ at a time $\tau = 60$.

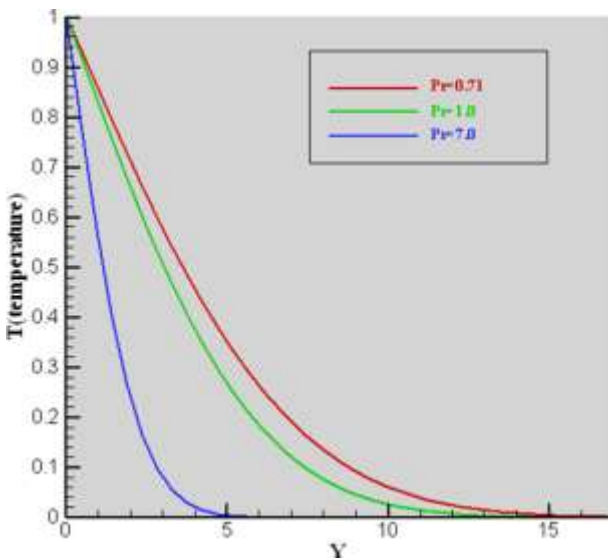


FIG. 9. Temperature profile for different values of Prandtl number P_r with fixed Eckert number $E_c = 0.01$ and Grashoff number $G_r = 1.0$ at a time $\tau = 10$.

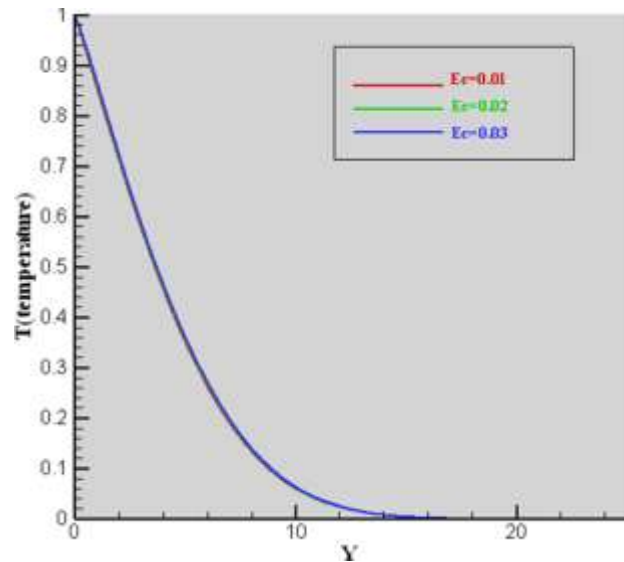


FIG. 11. Temperature profile for different values of Eckert number E_c with fixed Prandtl number $P_r = 0.71$ and Grashoff number $G_r = 1.0$ at a time $\tau = 10$.

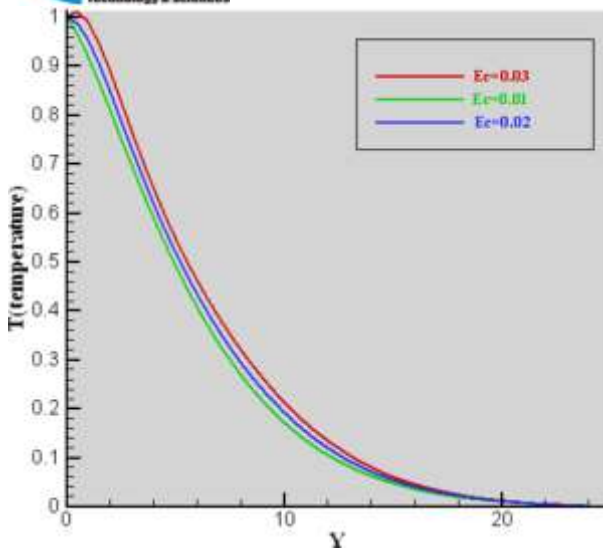


FIG. 12. Temperature profile for different values of Eckert number E_c with fixed Prandtl number $P_r = 0.71$ and Grashoff number $G_r = 1.0$ at a time $\tau = 60$.

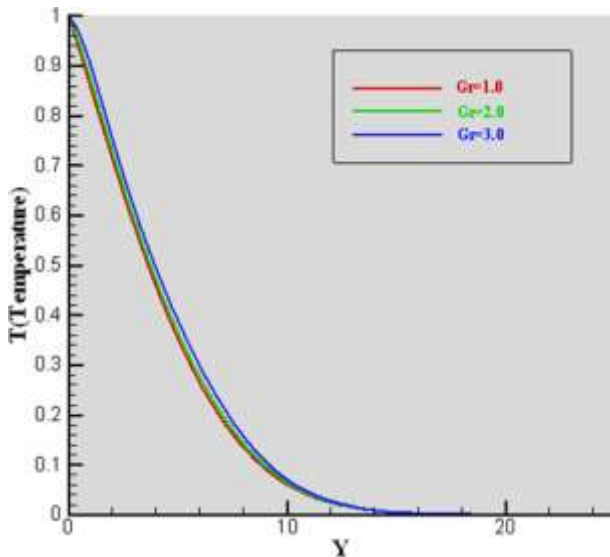


FIG. 13. Temperature profile for different values of Grashoff number G_r with fixed Eckert number $E_c = 0.01$ and Prandtl number $P_r = 0.71$ at a time $\tau = 10$.

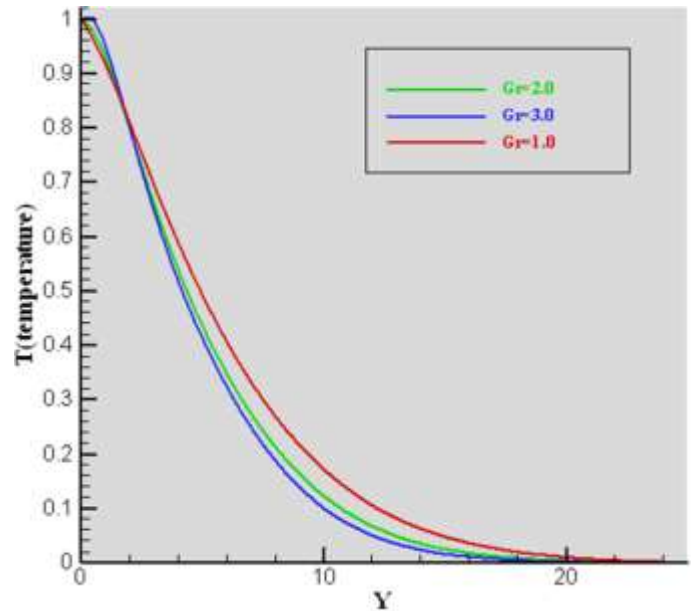


FIG. 14. Temperature profile for different values of Grashoff number G_r with fixed Eckert number $E_c = 0.01$ and Prandtl number $P_r = 0.71$ at a time $\tau = 60$.

5. CONCLUSION

Unsteady heat and mass transfer problem by free convection flow of an incompressible viscous fluid past an moving semi-infinite vertical plate under the different fluid pattern is taken into account. The plate as well as the fluid is considered at the same temperature and the concentration label is same. The results are discussed for different values of important parameters as Prandtl number, Eckert number, Grashof number. The important findings of these model are very effective for any kind of change of parameters. In all those parameters the Prandtl number (visible change for various fluid) as well as Grashof number are the main objective for our work, which has attracted the interest of any investigators in view of its important applications. Also the numerical solution gives clear information about the work.

APPENDIX

x : Co-ordinate along the plate surface

y : Co-ordinate normal to the plate surface

u : Velocity component in x -direction

v : Velocity component in y -direction

ρ : Local fluid density

μ : Co-efficient of viscosity

P_r : Molecular Prandtl number of the fluid

p : Hydrostatic pressure

ν : Kinematic viscosity

E_c : Eckert number

G_r : Thermal Grashof number

U, V : Dimensionless velocity of the fluid in the X, Y direction respectively.

t : Time

T : Temperature of the fluid in the boundary layer

T_{∞} : Ambient fluid temperature

T_w : Plate temperature

\bar{T} : Dimensionless Temperature

τ : Dimensionless time

Subscripts

w : Condition of the wall

∞ : Free Stream Condition.

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