

# Analysis of Shear Walls under Compression and Bending

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**Abstract—Design of shear walls is a complex procedure, especially if the cross section of the shear wall is not regular in shape. Shear walls are usually treated like cantilevers fixed at the foundation level. Normally, lift wells are designed as shear walls. The design of shear walls takes horizontal forces into account by shear and bending. Design for shear in the walls can be managed by computing the shear stress distribution over the cross-section and reinforcing appropriately. However, published literature is not available that would give the interaction of axial compression and bending moments on the section. This paper presents a methodology developed for generation of interaction diagrams for shear walls, thereby enabling a more accurate description of the capacity of the shear wall cross section under the effect of axial compression and lateral bending.**

**Keyword —Interaction Curves, RCC, Shear Walls.**

## 1. INTRODUCTION

Shear walls are a common choice for resisting horizontal forces in braced multistoreyed structures. RCC shear walls should ideally be located at corners or extreme ends, where side walls of the structure are nominated and designed as shear walls, and subsequently designed and constructed as RCC walls. However, the shear walls are commonly located as the walls of lift wells. These walls anyway do not need holes for windows, etc. and hence, prove to be a good choice. Unfortunately, lift wells are usually located towards the center of mass of the structure; thereby reducing the effectiveness of the walls. When more than one lift wells are selected to act as shear walls, the choice becomes ideal.

The shear walls are designed as cantilever shells (beams) fixed at the base or foundation. Thus, shear walls resist horizontal forces by bending about its neutral axis. The force system on the shear walls consist of horizontal loads and subsequently moments and shear forces along with the gravity loads acting downwards. To summarize, shear walls act as beam-columns. Shear over the sections may be very well taken care of, but the combination of axial compression along with the bending action must be taken into account while designing shear walls. The problem posed is more like design of RCC columns. Design of RCC columns having rectangular or circular shape is simple due to the fact that well established design tables and charts are available for description of

interaction of axial compression with the bending actions. However, for shear walls of non-rectangular shapes, such charts are not available. Further, it may not be practical to develop such charts as the shape and size of shear walls are most likely to change from structure to structure. Hence, it is deemed imperative to develop a method for computation of the interaction curves for commonly used shapes of shear walls. The framework applied to generation of interaction curves of common lift-well cross sectional shape with two lift wells is presented here. It must be noted that the shear walls may be fairly deep and hence first or higher order shear deformation theories may be required to be employed. However, only the classical (Euler-Bernoulli) theory of bending has been incorporated due to the fact that the algorithms for generation of interaction curves are based on equations presented in SP:16 [1].

Significant volume of research work is available for description of behavior of slender members under axial compression along with applied bending moment. Majority of research for is directed towards experimental and linear/non-linear analysis. A brief review of literature available for analysis of compression members is presented here.

Non-linear behavior of steel-concrete column sections subjected to bi-axial loading has been studied by de Sousa and Caldas [2]. They also compared various theories available for combining the influence of bi-axial bending moment on a cross-section under axial compression and developed a new capacity envelope. Razdolsky [3] developed a methodology to estimate buckling of laced columns. Chen [4] presented an iterative quasi-Newton for the rapid sectional analysis and design of short concrete-encased composite columns of arbitrary cross section subjected to biaxial bending. The stress resultants of concrete were evaluated by integrating the concrete stress-strain curve over the compression zone. El-Tawil S and Deierlein[5] proposed a method for determination of strength and ductility as a function of the percentage of steel. Narayanan and Kalyanraman[6] propose the design of column sections to be performed by giving considerations to the plastic capacity of the loaded column section. They also generate an interaction curve for a composite column section in order to determine the capacity of the section using the simplified design method suggested in the UK National Application Document for EC4. SP16 offers interaction curves for RCC column sections under axial

compression and uniaxial bending as per IS:456 (2000) [7]. However, the design curves as per design approach of IS:456 (2000) are not yet available for shear wall sections. This poses difficulties in estimating the capacity of shear walls considerably and subsequently designers usually tend to over-design them as the design process is empirical. The axial compression interacts with the flexural stress in a non-linear fashion, making it difficult to estimate the exact capacity of a given section. Thakkar [8] developed a framework for generation of interaction curves for encased columns, but of regular cross-section shapes. Interaction curves are vital for design of members under a combined action of axial compression and flexure. Empirical methods have been devised for designing shear walls, but the decisions about the amount of axial compression or bending moments that may be allowed on the section remain vague in absence of the interaction curves. Thus, it becomes imperative to obtain the interaction curves for shear wall sections also before commencing the design cycles in order to reduce the number of design cycles and achieve economy in design.

## 2. ANALYSIS

2.1. The approach adopted here follows the framework laid down by BIS. The strain variation adopted for the section under axial and bending actions when the neutral axis lies inside and outside the section is considered as shown in Fig 1.

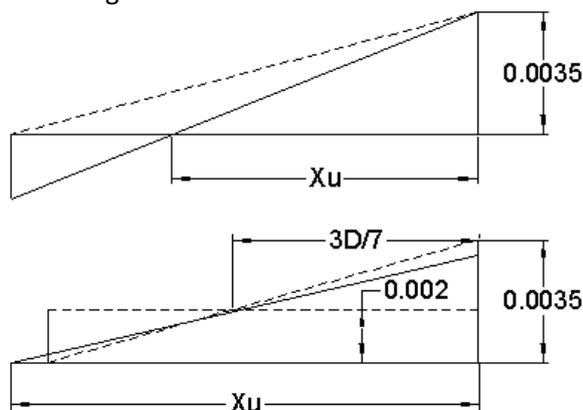


Fig 1: Strain Diagram

Linear strain distribution is adopted over the section conforming to the classical theory of bending as suggested by IS:456 (2000). Based on the strains at each point, the value of stress can be computed. The stress corresponding to strain for concrete is considered as per IS:456 (2000) and is reproduced in Fig 2.

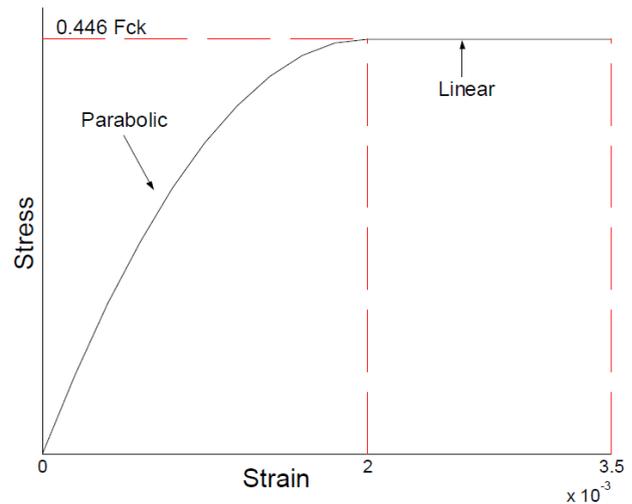


Fig 2: Concrete Stress Block

Stress varies parabolically with strain till a strain value of 0.002. Beyond that, the stress is constant and equal to  $0.446 F_{ck}$ . The fulcrum point of strain value 0.002 must be maintained for all positions of the neutral axis. When the neutral axis is inside the section, the strain values in the most compressed edge may exceed 0.0035. This imposes a limit on the position of neutral axis. On the other extreme, the neutral axis at infinity characterizes the condition of purely axial compression, which is analyzed without determining the position of neutral axis. SP16 suggests that it is sufficient to consider neutral axis  $1.5D$  beyond the edge of the section. This helps in setting a better initial guess of neutral axis position and speeds up the root finding algorithm. For each location of neutral axis 'Xu', the stress diagram can be obtained. Similarly, value of stresses at steel bar locations can also be computed, by considering the stress-strain diagram for steel reinforcements. For every position of neutral axis, the cross-section must be re-meshed. Since the shape of the section allows for rectangular mesh for any position of neutral axis, the meshing algorithm becomes extremely simple. Calculation of stress block parameters for a rectangular region becomes trivial. Hence, the equations for calculating stress block parameters have not been discussed here. The stress variation with respect to strain for the steel bars is considered to be linear. It is also assumed that the value of stress is averaged over the area of each bar, which corresponds to the value of strain at the centroid of the bar. The compressive stress in steel for a given strain is considered as per the recommendations of IS:456.

For each position of neutral axis, the section needs to be divided into rectangular segments. For each segment considered, the corresponding strain in the element at either edge is computed and subsequently the volume of stress block is obtained. The only care that needs to be taken while dividing the section in segments is that the segments must have edges parallel to the neutral axis, otherwise the calculation of stresses over the segment become cumbersome. The contribution of each element

of the section to the volume of stress block is added to the volume of stress block obtained for the section and the total axial load that can be carried is computed. The contribution of the element to the moment capacity is similarly computed. To compute each point on the interaction curve, the axial load 'Pu' is incremented from 0 to 'Pcr', where 'Pcr' is the total axial compressive load capacity of the column in absence of bending moments. For each value of intermediate 'Pu', the value of 'Xu' must be determined, which in turn dictates the strain variation over the section and thus the stress variation. The stress variation is used to compute the value of axial load and bending capacity of the section. Thus, the value of 'Xu' must be calculated by non-linear iterative procedures. Nonlinear solution has been performed here by Brent algorithm.

2.2. The section considered for analysis is shown in Fig 3. The section geometry is adopted based on the regular geometry of lift wells used commonly in multistoreyed RCC framed structures.

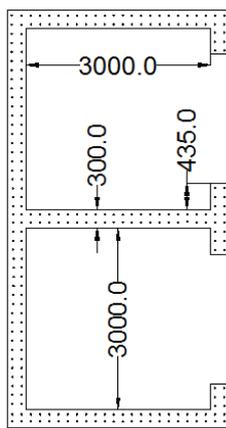


Fig 3: Schematic of Shear Wall Section

Interaction curves depend upon the direction about which the bending action is considered. This is primarily due to the fact that the moment capacity depends on the moment of inertia about any given axis. Hence, the section has different moment resisting capacity in both x and y directions. Further, for unsymmetrical sections the moment resistance also varies based on the sign of the applied moment. Hence, we would ideally get four different interaction diagrams for any unsymmetrical section. Interaction curves along all four axes (positive x, negative x, positive y and negative y) have been generated and reported. It is clear from the plot that there are four interaction curves for the section. Of course, the curve to be considered depends upon the design problem at hand. Since these interaction curves have been generated for the particular section, the values are not normalized for section parameters, but are reported in force and moment units. Further, strength envelope and capacity surface have been generated and reproduced

here. The interaction curves generated for the section are shown in Fig 4.

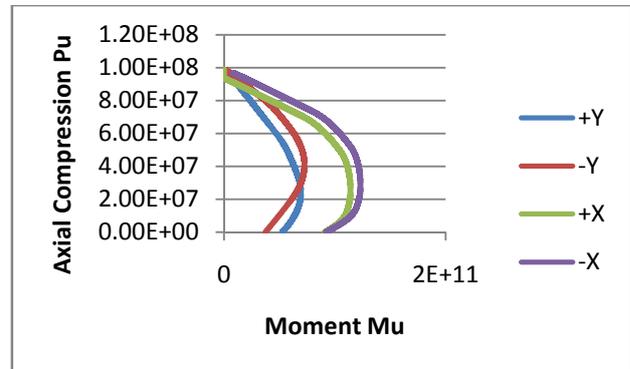


Fig 4: Interaction Curves

The interaction curves presented in SP:16 are normalized with the dimensions of the cross-section considered. While such normalization makes the charts generalized for any cross-section having the shape considered while generation, such normalization is not possible here because of the irregularity in shape of the cross section. Another very important aspect of column capacity is the strength envelope. The strength envelope can be said to be the envelope obtained for a given value of axial compression Pu with varying directions of neutral axis. Thus, strength envelope can also be said to be a horizontal cross section of a 3-D capacity or failure surface, whereas interaction curves can be visualized as vertical cross section of the 3-D capacity surface. The strength envelopes for various values of Pu obtained for the section considered are shown in Fig 5. IS:456 (2000) considers a power law for combination of the moments from both sides. This method may not be required here as the strength envelopes are generated by rotating the neutral axis before computing the moment capacity. The full 3D capacity surface with neutral axes rotated 360° for the section considered is shown Fig 6.

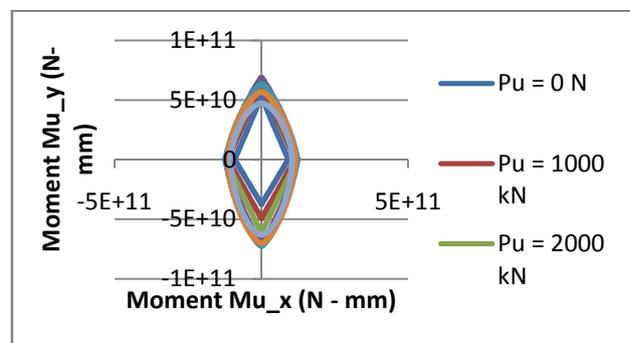
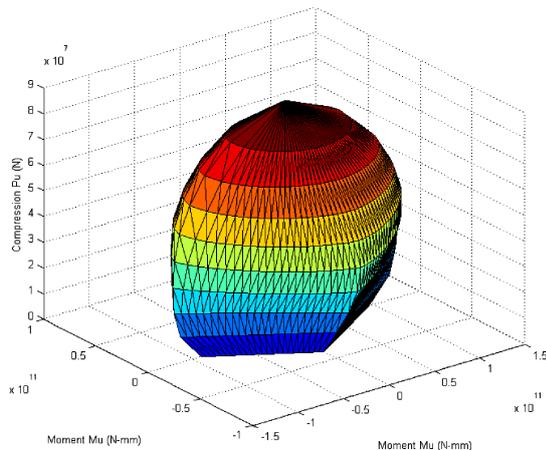


Fig 5: Strength Envelopes



**Fig 6: Capacity Surface**

### 3. VALIDATION

For validation purpose, interaction curves generated by the methodology shown above for rectangular cross-section with all around placement of bars have been compared with the standard results published in SP:16. The curves match with extreme precision. This is because the equations incorporated in the above algorithm are the same as those used for the standard results published in SP:16. The only variation considered here is the difference in shape. Thus, the methodology and algorithm developed for division of the section and subsequent calculation of stress blocks can be considered to be appropriate.

### 4. CONCLUSION

A methodology developed for generation of interaction curves and strength envelope of shear wall sections has been presented here. Interaction curves and strength envelopes have been presented. 3-D capacity surface has been reproduced, which indicate the strength of the section in each direction. The methodology appears to be promising and tries to give a more robust technique for design of RCC shear walls, thereby making the design process of shear walls more definitive.

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Dr. Bhairav K. Thakkar has obtained his Ph.D. in structural engineering from Indian Institute of Science, Bangalore. He has two year post-doctoral research experience from Eindhoven University of Technology, The Netherlands. He is a founder director of the software and structural consultancy company “Fairylend Technology Pvt Ltd”. He is former Associate Professor and Head of Civil Engineering department, Parul Institute of Engineering and Technology, Waghodia, Vadodara. Currently, he works as visiting faculty in department of Applied Mechanics and Structural Engineering, M. S. University of Baroda, Vadodara.